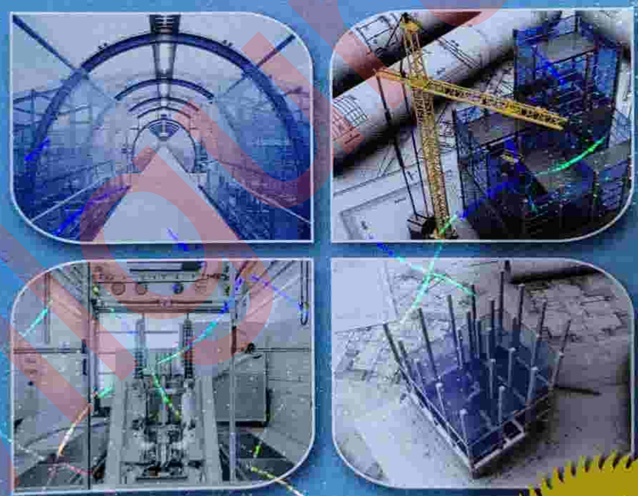
Quantum Series

ENGINEERING

QUANTUM Series

Semester - 4 Civil Engineering

Hydraulic Engineering and Machines



Session 2019-20 Even Semester

- Topic-wise coverage of entire syllabus in Question-Answer form.
- Short Questions (2 Marks)

Includes solution of following AKTU Question Papers:
2014-15 • 2015-16 • 2016-17 • 2017-18 • 2018-19

QUANTUM SERIES

For
B.Tech Students of Second Year
of All Engineering Colleges Affiliated to
Dr. A.P.J. Abdul Kalam Technical University,
Uttar Pradesh, Lucknow
(Formerly Uttar Pradesh Technical University)

Hydraulic Engineering and Machines

By

Vikas Yadav

Quantum Page

QUANTUM PAGE PVT. LTD.
Ghaziabad ■ New Delhi

PUBLISHED BY : Apram Singh
Quantum Page Pvt. Ltd.
Plot No. 59/2/7, Site - 4, Industrial Area,
Sahibabad, Ghaziabad-201 010

Phone : 0120 - 4160479

Email : pagequantum@gmail.com **Website:** www.quantumpage.co.in

Delhi Office : 1-6590, East Rohtas Nagar, Shahdara, Delhi-110032

Hydraulic Engineering and Machines (CE : Sem-4)

1st Edition : 2013-14
2nd Edition : 2014-15
3rd Edition : 2015-16
4th Edition : 2016-17
5th Edition : 2017-18
6th Edition : 2018-19
7th Edition : 2019-20 (Thoroughly Revised Edition)

Price: Rs. 120/- only

Printed at : Mayank Enterprises, Delhi-110093.

CONTENTS

KCE 403 : Hydraulic Engineering [illegible]

1
UNIT

Flow Through Open Channel

CONTENTS

PART-1

Introduction : Basic Concepts of Free Surface Flows, Velocity and Pressure Distribution, Mass, Energy and Momentum Principle for Prismatic and Non-Prismatic Channel's, Critical, Subcritical and Supercritical Types of Flows.

CONCEPT OUTLINE

Prismatic and Non-prismatic Channels : A channel with constant cross-sectional shape and size and also the bottom slope is termed as prismatic channel. All natural channels generally have varying cross-sections and consequently are non-prismatic.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.1. **Differentiate between pipe flow and open channel flow.**

Answer

S. No.	Description	Open Channel Flow	Pipe Flow
1.	Pressure	Liquid flowing through open channel is always at atmospheric pressure.	Liquid flowing through pipe is always at a higher pressure than atmospheric pressure.
2.	Slope	For open channel flow liquid flows under gravity, thus slope has to be provided.	The pipe flow does not require slope.
3.	Velocity	For open channel flow maximum velocity occurs little below the free surface.	For pipe flow maximum velocity occurs at centre of pipe.
4.	Hydraulic gradient line	Hydraulic gradient line coincides with water surface.	Hydraulic gradient line does not coincide with water surface.

5.	Shape of channel	The shape of open channel may be circular, trapezoidal, rectangular, triangular, etc.	Generally the shape of a pipe is circular.
6.	Surface roughness	Hydraulic roughness depends upon channel parameter, depth of flow.	Roughness co-efficient depends upon material of pipe.

Que 1.2. Define different types of flow. **AKTU 2017-18, Marks 02**

Answer

Flow in open channel can be classified on the following basis :

1. **Classification Based on Flow Characteristics :**

   i. Steady flow. ii. Unsteady flow.

S. No.	Steady Flow	Unsteady Flow
1.	Steady flow is defined as the type of flow in which the fluid characteristics like velocity, pressure, density, etc., at a point do not change with time.	Unsteady flow is that type of flow, in which the velocity, pressure or density at a point changes with respect to time.
2.	Thus for steady flow, $\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} = 0$, $\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0$, $\left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$	Thus for unsteady flow, $\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$, $\left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$ etc.

2. **Classification Based on Flow and Channel Characteristics :**

   i. Uniform flow. ii. Non uniform flow.

S. No.	Uniform Flow	Non Uniform Flow
1.	Uniform flow is defined as the type of flow in which the velocity at any given time does not changes with respect to space (*i.e.*, length of direction of the flow).	Non uniform flow is defined as the type of flow in which the velocity at any given time changes with respect to space.
2.	For uniform flow $\left(\frac{\partial v}{\partial s}\right)_{t = \text{constant}} = 0$	For non-uniform flow $\left(\frac{\partial v}{\partial s}\right)_{t = \text{constant}} \neq 0$

Non-uniform flow can be further classified as follows :

a. **Gradually Varied Flow :** If the depth of flow in a channel changes gradually over a long length of channel then the flow is said to be gradually varied flow.

b. **Rapidly Varied Flow :** If the depth of flow changes suddenly over a small length of channel then the flow is said to be rapidly varied flow.

c. **Spatially Varied Flow :** If some flow is added or subtracted from the system the resultant flow is known as spatially varied flow.

3. **Classification Based on Reynold's Number :**

   For open channel Reynold's number is given as

$$R_e = \frac{\rho v R}{\mu}$$

where, R_e = Reynold's number, ρ = Density of water.
μ = Viscosity of water, v = Mean velocity of flow of water.

On the basis of Reynold's number open channel flow can be classified as :

i. **Laminar Flow :** For laminar flow, $R_e < 500$.
ii. **Transitional Flow :** For transitional flow, $500 < R_e < 2000$.
iii. **Turbulent Flow :** For turbulent flow $R_e > 2000$.

4. **Classification Based on Froude Number :**

   On the basis of Froude number open channel flow is classified as :

i. **Sub-critical Flow :** For sub critical flow, $F_r < 1$.
ii. **Critical Flow :** For critical flow, $F_r = 1$.
iii. **Super-critical Flow :** For supercritical flow, $F_r > 1$.

Que 1.3. **Discuss the various geometrical parameters used in designing of a channel.**

Answer

Following are the various geometrical parameters used in designing of a channel :

1. **Depth of Flow (y) :** It is the vertical distance of the lowest point of a channel section from the free surface.
2. **Top Width (*T*) :** It is the width of the channel section at the free surface.
3. **Wetted Area (*A*) :** It is the cross-section area of the flow normal to the direction of flow.
4. **Wetted Perimeter (*P*) :** It is the length of the channel boundary in contact with the flowing water at any section.
5. **Hydraulic Radius or Hydraulic Mean Radius (*R*) :** It is the ratio of the wetted area to its wetted perimeter.

$$R = A/P$$

6. **Hydraulic Depth Hydraulic Mean Depth (D) :** It is the ratio of the wetted area to the top width T.

$$D = A/T$$

7. **Section Factor (Z) :**

i. For critical flow computation, $Z = A\sqrt{D} = (A^3/T)^{1/2}$ $\quad (\because \ D = A/T)$

ii. For uniform flow computation, $Z = AR^{2/3} = (A^{5/3}/P)$ $\quad (\because \ D = A/P)$

iii. At critical flow condition, $y = y_c$ and $Z_c = A_c\sqrt{\dfrac{A_c}{T_c}} = \dfrac{Q}{\sqrt{g}}$ ...(1.3.1)

iv. From the eq. (1.3.1), if Z is the section factor for any depth of flow y, then

$$Q_c = \sqrt{g} \times Z$$

Where Q_c represents the discharge that would make the depth y critical and is known as the critical discharge.

Que 1.4. **Draw typical velocity profile at a vertical section of an open channel.**

Answer

1. The presence of corners and boundaries in an open channel causes the velocity vectors of the flow to have components not only in the longitudinal and lateral direction but also in normal direction to the flow.
2. In a macro-analysis only longitudinal component, v_x is of major concern. The other two components being small are ignored and v_x is designated as v.
3. The velocity distribution in a channel is dependent on the geometry of the channel. Fig. 1.4.1(a) and Fig. 1.4.1(b) show contours of equal velocity for a natural and rectangular channel respectively.

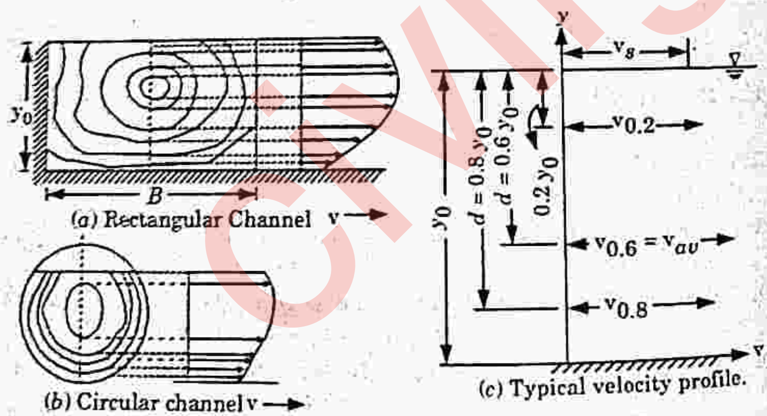


Fig. 1.4.1. Velocity distribution in open channels.

4. The velocity 'v' is zero at solid boundaries and gradually increases with distance from the boundary. The maximum velocity of the cross-section occurs at a certain distance below the free surface.
5. This dip of the maximum velocity is due to secondary currents and is a function of the aspect ratio (ratio of depth to width) of the channel.
6. For a deep narrow channel, the location of the maximum velocity point will be much lower from the water surface than for a wider channel of the same depth.
7. A typical velocity profile at a section in a plane normal to the direction of flow is shown in Fig. 1.4.1(c).
8. In figure the average velocity at any vertical v_{av}, occurs at a level of 0.6 y_0 from the free surface. It is found that,

$$v_{av} = \frac{v_{0.2} + v_{0.8}}{2}$$

where, $v_{0.2}$ = Velocity at a depth of $0.2y_0$ from the free surface.
$v_{0.8}$ = Velocity at a depth of $0.8y_0$ from the free surface.
y_0 = Depth of flow

9. The surface velocity v_s is related to the average velocity v_{av} as

$$v_{av} = kv_s$$

where k is a coefficient with a value between 0.8 and 0.95.

Que 1.5. **Deduce the basic equation of continuity for unsteady open channel flow.**

Answer

1. In the unsteady flow of incompressible fluids, if we consider a reach of the channel, the continuity equation states that the net discharge going out of all the boundary surfaces of the reach is equal to rate of depletion of the storage within it.
2. In Fig. 1.5.1, if $Q_2 > Q_1$, more flow goes out than what is coming into section 1.
3. The excess volume of outflow in a time Δt is made good by the depletion of storage within the reach bounded by sections 1 and as a result of this the water surface will start falling.
4. If Δx is the distance between sections 1 and 2, then

$$Q_2 - Q_1 = (\partial Q / \partial x)\,\Delta x$$

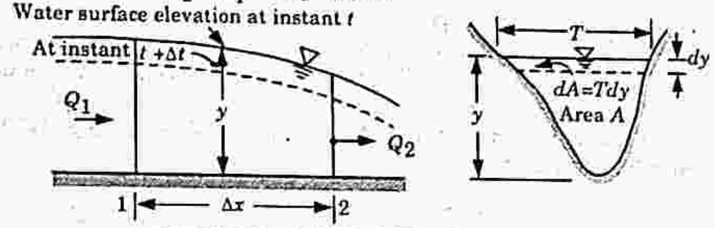


Fig. 1.5.1. Definition sketch of unsteady flow.

5. The excess volume rate of flow in a time $\Delta t = (\partial Q / \partial x)\,\Delta x\,\Delta t$

6. If the top width of the channel at any depth y is T, $T = \partial A / \partial y$. The storage volume at depth $y = A \times \Delta x$
7. The rate of decrease of storage $= -\Delta x \dfrac{\partial A}{\partial y}\dfrac{\partial y}{\partial t} = -T\Delta x \dfrac{\partial y}{\partial t}$
8. The decrease in storage in time $\Delta t = \left(-T\Delta x \dfrac{\partial y}{\partial t}\right)\Delta t$
9. By continuity, $\dfrac{\partial Q}{\partial x}\Delta x \Delta t = -T\dfrac{\partial y}{\partial t}\Delta x \Delta t \Rightarrow \dfrac{\partial Q}{\partial x} + T\dfrac{\partial y}{\partial t} = 0$

   This is the basic equation of continuity for unsteady open channel flow.

Que 1.6. **Deduce basic equation of continuity for spatially varied open channel flow.**

Answer

1. The channel under consideration as shown in Fig. 1.6.1 has no limitation on geometry of cross section or alignment.

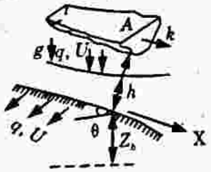


Fig. 1.6.1. Schematic drawing of spatially varied channel flow.

2. The liquid flowing in the channel is incompressible, viscous, and nonhomogeneous; *i.e.*, both the density ρ and dynamic viscosity μ of the fluid may vary from point to point, but the density and viscosity of an infinitesimal incompressible element remain constant with time.
3. The lateral flow through the free surface such as rainfall or infiltration or seepage inflow can vary with both space and time and their fluid properties can also be nonhomogeneous but necessarily incompressible.
4. For a point (more precisely, an infinitesimal unit volume) in a turbulent unsteady flow the continuity equation is

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho \bar{u}_i}{\partial x_i} = 0 \qquad ...(1.6.1)$$

where, t = Time,

u_i = Local velocity component of the fluid along the x_i direction; and the bar represents temporal averaging over turbulent fluctuation.

5. For a channel cross section with an area A as shown in Fig. 1.6.2, the boundary condition is.

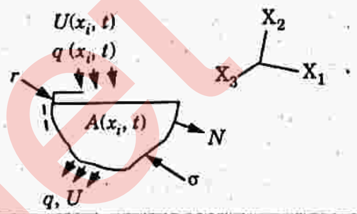


Fig. 1.6.2. Definition sketch of a channel cross section.

$$\left[G\frac{\partial A}{\partial t} + G\bar{u}_i\frac{\partial A}{\partial x_i}\right] = \int_\sigma Gq\, d\sigma \qquad ...(1.6.2)$$

Where, σ = Perimeter bounding A,

G = Any continuous scalar quantity under consideration at σ, e.g., $G = \rho$ for mass conservation, and

q = Rate of lateral flow into the channel per unit length of σ, having a dimension of length per unit time and being positive for inflow.

6. The continuity equation for a channel cross section can be obtained by integrating the point continuity equation over A. By applying the Leibnitz rule, integration of eq. (1.6.1), we get

$$\int_A\left(\frac{\partial \rho}{\partial t} + \frac{\partial \rho \bar{u}_i}{\partial x_i}\right)dA = \int_A \rho dA - \left[\rho\frac{\partial A}{\partial t}\right]_\sigma + \frac{\partial}{\partial x_i}\int_A \rho\bar{u}_i\, dA - \left[\rho u_i \frac{\partial A}{\partial t}\right]_\sigma = 0 \qquad ...(1.6.3)$$

or, with the boundary condition, equation (1.6.2) with $G = \rho$,

$$\frac{\partial}{\partial t}\int_A \rho\, dA + \frac{\partial}{\partial x_i}\int_A \rho\bar{u}_i\, dA - \int_\sigma \rho q\, d\sigma = 0 \qquad ...(1.6.4)$$

which is the continuity equation in integral form for a channel cross section.

7. If the cross sectional mean fluid density ρ_a is defined as,

$$\rho_a = \frac{1}{A}\int_A \rho\, dA \qquad ...(1.6.5)$$

then eq. (1.6.4) can be written in one-dimensional form as,

$$\frac{\partial}{\partial t}(\rho_a A) + \frac{\partial}{\partial x_i}(\lambda_{ki}\,\rho_a\, A\, v_i) = \int_\sigma \rho q\, d\sigma \qquad ...(1.6.6)$$

in which the mean velocity component of the cross section, v_i, is defined as,

$$v_i = \frac{1}{A}\int_A \bar{u}_i\, dA \qquad ...(1.6.7)$$

and the density correction factor λ is defined as,

$$\lambda_{ki}\,\rho_a A v_i = \int_A \rho\bar{u}_i\, dA \qquad ...(1.6.8)$$

8. If the channel cross section is taken such that its normal is along the direction $i = 1$, then, with $\lambda_{11} = \lambda$, the continuity equations is one-dimensional eq. (1.6.6) can be simplified as

$$\frac{\partial}{\partial t}(\rho_s A) + \frac{\partial}{\partial x}(\lambda \rho_s A v_1) = \int_c \rho q \, d\sigma$$

PART-2

Critical Depth, Concepts of Specific Energy and Specific Force.

CONCEPT OUTLINE

Critical Depth : Critical depth is defined as that depth of flow of water at which the specific energy is minimum.

Specific Energy : Specific energy of flowing liquid is defined as energy per unit weight of the liquid with respect to the bottom of the channel.

Specific Force : It is the sum of the pressure force and momentum flux per unit weight of the fluid at a section.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.7. What do you understand by specific energy for a flow in open channel ? Draw the specific energy diagram and describe its various characteristics.

AKTU 2014-15, Marks 10

Answer

A. Specific Energy :

1. The total energy of a channel flow referred to a datum is given by,

$$H = Z + y\cos\theta + \alpha(v^2/2g)$$

2. If the datum coincides with the channel bed at the section, the resulting expression is known as specific energy and is denoted by E. Thus

$$E = y\cos\theta + \alpha(v^2/2g)$$

When, $\cos\theta = 1$ and $\alpha = 1$

$$E = y + (v^2/2g)$$

3. Hence specific energy of flowing liquid is defined as energy per unit weight of the liquid with respect to the bottom of the channel.

B. Specific Energy Curve :

1. It is defined as the curve which shows the variation of specific energy with depth of flow.

2. The specific energy of a flowing liquid,

$$E = y + \frac{v^2}{2g} = E_p + E_k$$

where, E_p = Potential energy of flow = y

E_k = Kinetic energy of flow = $\frac{v^2}{2g}$

3. Consider a rectangular channel in which a steady but non uniform flow is taking place.

4. Let

Q = Discharge through the channel.

b = Width of the channel.

y = Depth of flow.

q = Discharge per unit width.

4. Then $q = \frac{Q}{\text{width}} = \frac{Q}{b}$ = Constant [$\because$ Q and b are constant]

5. Flow velocity, $v = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{b \times y} = \frac{q}{y}$

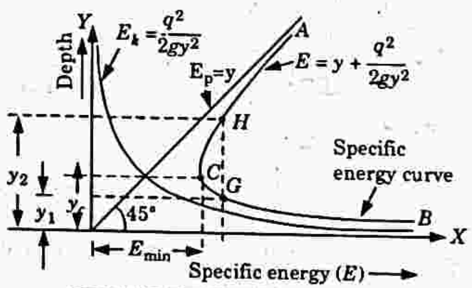


Fig. 1.7.1. Specific energy curve.

6. Specific energy, $E = y + \frac{q^2}{2gy^2} = E_p + E_k$ ...(1.7.1)

7. Eq. (1.7.1) gives the variation of specific energy (E) with depth of flow (y). Then a graph between specific energy (along X-axis) and depth of flow, y (along Y-axis) may be plotted.

C. Characteristics of Specific Energy Curve :

1. The curve is asymptotic to abscissa and the line OA, *i.e.*, $E = y$.
2. As the depth increases from low depth, *i.e.*, from supercritical depth, E value decreases upto a minimum value at point C.
3. E value increases when depth increases after the depth at C.
4. At the point of minimum specific energy, *i.e.*, at point C, the depth of flow is critical (y_c).
5. For two different depths y_1 and y_2 same specific energy ($E_1 = E_2$) is obtained and these two depths at same specific energy are called alternate depths.

6. Zones below or above OA and specific curve are the zones of super critical and subcritical zones.
7. If the discharge increases or decreases, critical depth will also increases or decreases, respectively and specific energy curve also follows their alignment towards the right or left.

Que 1.8. **Derive the expression for specific force in a rectangular channel section.**

Answer

1. The steady state momentum equation is given by
$$\Sigma F = F_1 - F_2 - F_3 + F_4 = M_2 - M_1$$
in which F_1 and F_2 are pressure forces acting on the control surfaces.
2. F_3 is tangential force acting on the bed.
3. F_4 is body force, *i.e.*, the component of the weight of the fluid in the longitudinal direction.

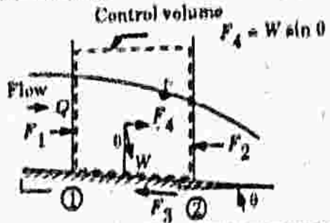


Fig. 1.8.1. Definition sketch for the momentum equation.

4. $M_1 = \beta_1 \rho Q v_1$ = Momentum flux entering the control volume.
$M_2 = \beta_2 \rho Q v_2$ = Momentum flux leaving the control volume.
5. If F_3 and F_4 are both zero, then $F_1 - F_2 = M_2 - M_1$
$$\Rightarrow \quad F_1 + M_1 = F_2 + M_2$$
6. Denoting $\dfrac{1}{\gamma}(F + M) = P_s \quad \Rightarrow \quad (P_s)_1 = (P_s)_2$ ...(1.8.1)
7. The term P_s is known as the specific force and represents the sum of the pressure force and momentum flux per unit weight of the fluid at a section.
8. Eq. (1.8.1) states that the specific force is constant in a horizontal, frictionless channel.

Que 1.9. **Show that for a rectangular channel carrying constant discharge, the specific force is minimum, when the depth is critical.**

OR

Explain the specific energy concept and prove the critical flow condition for all type of channel.

AKTU 2018-19, Marks 07

Answer

Specific Energy : Refer Q. 1.7, Page 1-9B, Unit-1.

Critical Flow Condition :

1. For a given discharge the condition for minimum specific force can be obtained by differentiating equation of specific force with respect to y and then considering it as $dF/dy = 0$.
$$F = Q^2/gA + A\bar{z} \qquad ...(1.9.1)$$
2. Differentiate of Eq. (1.9.1) with respect to, we get
$$\frac{dF}{dy} = -\frac{Q^2}{gA^2}\frac{dA}{dy} + \frac{d(A\bar{z})}{dy} = 0 \qquad ...(1.9.2)$$
3. Since Q is constant and both A and $\bar{z}$ are the functions of y. As shown in Fig. 1.9.1 for a change dy in the depth, the corresponding change $d(A\bar{z})$ in the moment of the cross-sectional area about the free surface may be expressed as, $\dfrac{d(A\bar{z})}{dy} = \left(A(\bar{z} + dy) + T dy \dfrac{dy}{2}\right) - A\bar{z}$

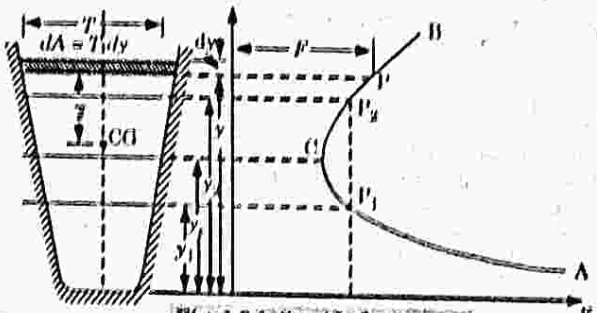


Fig. 1.9.1. Specific-force curve.

4. Neglecting smaller term, $\dfrac{T(dy)^2}{2}$ we get
$$d(A\bar{z}) = A dy$$
5. Substituting this value of $d(A\bar{z})$ in eq. (1.9.2), we get
$$\therefore \quad \frac{dF}{dy} = -\frac{Q^2}{gA^2}\frac{dA}{dy} + \frac{A dy}{dy} = 0$$
6. Again since $(dA/dy) = T$, the above equation reduces
$$\text{or} \quad \frac{Q^2 T}{gA^3} = 1 \qquad \left[\because \frac{dA}{dy} = T\right]$$
7. The above condition is critical flow condition. Thus it can be said that at critical depth, specific force attains minimum value.

Answer

Given : Velocity of flow, v = 2 m/sec, Depth of flow, y = 1.25 m
Apex angle = 60°
To Find : Types of flow, Specific energy.

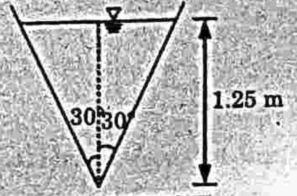


Fig. 1.13.1.

1. Froude number, $$F_r = \frac{\sqrt{2}\,v}{\sqrt{gy}} = \frac{\sqrt{2}\times 2}{\sqrt{9.81\times 1.25}} = 0.81 < 1$$

   Hence, flow is subcritical.

2. Specific energy, $$E = y + \frac{v^2}{2g} = 1.25 + \frac{2^2}{2\times 9.81} = 1.45 \text{ m}$$

Que 1.14. **Show that in rectangular channel maximum discharge occurs when the flow is critical for a given value of specific energy.**

AKTU 2015-16, Marks 10

Answer

1. The specific energy at any section of a channel is given by,
$$E = y + \frac{v^2}{2g}, \quad \text{where, } v = \frac{Q}{A} = \frac{Q}{B\times y}$$
$$\therefore \quad E = y + \frac{Q^2}{2gB^2y^2} \Rightarrow Q^2 = (E-y)\,2gB^2y^2$$
$$Q = \sqrt{(E-y)\,2gB^2y^2} = B\sqrt{2g(Ey^2-y^3)}$$
2. For maximum discharge, the expression $(Ey^2 - y^3)$ should be maximum.
$$\text{or} \quad \frac{d}{dy}(Ey^2-y^3) = 0 \Rightarrow 2Ey - 3y^2 = 0 \Rightarrow E = \frac{3}{2}y$$
Also $$E_{min} = \frac{3}{2}y_c, \quad \text{where } y_c = \text{Critical depth.}$$
Hence the condition for maximum discharge for a given value of specific energy is that the depth of flow should be critical.

Que 1.15. **A rectangular channel 2.5 m wide has a specific energy of 1.50 m when carrying a discharge of 6.48 m³/sec. Calculate the alternate depths and corresponding Froude numbers.**

Answer

Given : Width of channel, B = 2.5 m, Specific energy, E = 1.5 m
Discharge, Q = 6.48 m³/sec
To Find : Alternate depth, Froude numbers.

1. Specific energy is given by, $$E = y + \frac{v^2}{2g}$$
Velocity of flow, $$v = \frac{Q}{By} = \frac{6.48}{2.5y}$$
$$\therefore \quad 1.5 = y + \frac{(6.48)^2}{(2.5y)^2}\times\frac{1}{2\times 9.81}$$
$$\text{or} \quad y^3 - 1.5y^2 + 0.3425 = 0$$
Alternate depths, $y_1 = 1.29$ m and $y_2 = 0.62$ m
2. Froude number corresponding to y_1,
$$F_{r1} = \frac{v}{\sqrt{gy_1}} = \frac{6.48}{2.5\times 1.29\times\sqrt{9.81\times 1.29}} = 0.564$$
3. Froude number corresponding to y_2,
$$F_{r2} = \frac{6.48}{2.5\times 0.62\times\sqrt{9.81\times 0.62}} = 1.695$$

Que 1.16. **A trapezoidal channel with a base width of 6 m and side slopes of 2 horizontal to 1 vertical conveys water at 17 m³/sec with a depth of 1.5 m. Is the flow situation sub or super critical ?**

AKTU 2016-17, Marks 10

Answer

Given : Base width, B = 6 m, Side slope = 2 H:1V,
Discharge, Q = 17 m³/sec, Depth, y = 1.5 m
To Find : Flow condition.

1. Area of the trapezoidal channel, $A = (B + my)y$
$= (6 + 2 \times 1.5) \times 1.5 = 13.5 \text{ m}^2$
2. Top width of channel, $T = (B + 2\,my) = (6 + 2 \times 2 \times 1.5) = 12$ m
3. Velocity of flow, $$v = \frac{Q}{A} = \frac{17}{13.5} = 1.26 \text{ m/sec}$$
4. Froude number for trapezoidal section,
$$F_r = \frac{v}{\sqrt{g(A/T)}} = \frac{1.26}{\sqrt{9.81\times(13.5/12)}} = 0.38 < 1$$
5. Since the value of Froude number is less than 1. The flow will be subcritical.

Que 1.18. **Express Chezy's formula in terms of Darcy-Weisbach friction factor.**

Answer

1. For pipe flow, the Darcy-Weisbach equation is given by,

$$h_f = f\frac{L}{D}\times\frac{v^2}{2g}$$

where, h_f = Head loss due to friction in pipe of diameter D.
L = Length of pipe.
f = Darcy-Weisbach friction factor.

2. An open channel can be considered as conduit cut into two parts.

For circular section, $R = \frac{A}{P} = \frac{\pi D^2}{4\pi D} = \frac{D}{4} \Rightarrow D = 4R$

$$\therefore \quad h_f = f\frac{L}{4R}\frac{v^2}{2g} \Rightarrow v = \sqrt{\frac{8g}{f}}\times\sqrt{R}\times\sqrt{\frac{h_f}{L}} \quad ...(1.18.1)$$

$$\frac{h_f}{L} = S_f = \text{Slope of energy line.}$$

4. But in uniform flow, $S_f = S_0$

Chezy's formula is given by, $v = C\sqrt{RS_0}$ ...(1.18.2)

5. Comparing eq. (1.18.1) and (1.18.2), we get

$$C = \sqrt{\frac{8g}{f}}, \quad S_0 = S_f = \sqrt{\frac{h_f}{L}}$$

Que 1.19. **Compare Chezy's formula and Manning's formula and give relation between friction factor and Manning's coefficient.**

Answer

1. According to Chezy's equation, $v = C\sqrt{RS_0}$ ...(1.19.1)
2. According to Manning's equation, $v = \frac{1}{n}R^{2/3}S_0^{1/2}$ ...(1.19.2)
3. Equating both eq. (1.19.1) and (1.19.2), we have

$$\frac{1}{n}R^{2/3}S_0^{1/2} = C\sqrt{RS_0} \Rightarrow C = \frac{1}{n}R^{1/6}$$

$$\sqrt{\frac{8g}{f}} = \frac{1}{n}R^{1/6} \Rightarrow f = \left(\frac{n^2}{R^{1/3}}\right)8g \qquad \left(\because C = \sqrt{\frac{8g}{f}}\right)$$

Que 1.20. **Show by using Manning's formula that the average boundary shear stress is given by, $\tau_0 = \frac{\rho g n^2 v^2}{R^{1/3}}$**

Answer

Consider a small section of an open channel. The forces acting on the free body of water in the direction of flow are as follows :

1. Forces of hydrostatic water pressure F_1 and F_2, acting on the two ends of the free body.

$F_1 = F_2$ (Depth of water at section (1) and (2) are same.)

2. Component of weight of water in the direction of flow = $wAL \sin\theta$.

where, w = Specific weight of water.
A = Wetted cross-sectional area of channel.
θ = Angle of inclination of channel bottom with horizontal.

3. The resistance of flow exerted by wetted surface of channel = $PL\tau_0$

where, P = Wetted perimeter of the channel.
τ_0 = Average shear stress at the channel boundary.

4. According to Newton's second law of motion,

$$wAL\sin\theta - PL\tau_0 = 0$$

$$\therefore \quad \tau_0 = \frac{wAL\sin\theta}{PL} = w\left(\frac{A}{P}\right)\sin\theta$$

$$\tau_0 = wRS_0 = \rho gRS_0 \quad ...(1.20.1)$$

5. From Manning's equation, $v = \frac{1}{n}R^{2/3}S_0^{1/2} \Rightarrow S_0 = \frac{v^2n^2}{R^{4/3}}$

6. From eq. (1.20.1), we get $\tau_0 = \frac{\rho gRv^2n^2}{R^{4/3}} = \frac{\rho gv^2n^2}{R^{1/3}}$

Que 1.21. **A trapezoidal channel with side slopes of 2(H) : 1(V) has to be designed to carry 20 m³/sec at slope of 1/5500. Determine the depth of flow. Bottom width 3.0 m and Manning's coefficient = 0.015.**

Answer

Given : Discharge, $Q = 20$ m³/sec, Bad slope, $S_0 = 1/5500$
Bottom width, $B = 3$ m, Manning's coefficient, $n = 0.015$
To Find : Normal depth of flow.

1. Let normal depth = y_0

Area, $A = (3 + 2y_0)y_0$

2. Wetted perimeter, $P = 3 + (2\sqrt{2^2+1})\, y_0 = 3 + 4.47\, y_0$

3. Hydraulic mean radius, $R = \frac{A}{P} = \frac{(3+2y_0)y_0}{(3+4.47y_0)}$

4. The section factor is given by, $AR^{\frac{2}{3}} = \frac{Qn}{\sqrt{S_0}}$

$$(3+2y_0)y_0 \times \left[\frac{(3+2y_0)y_0}{(3+4.47y_0)}\right]^{2/3} = \frac{20 \times 0.015}{\sqrt{0.0001818}} = 22.25 \quad ...(1.21.1)$$

On solving eq. (1.21.1) normal depth, $y_0 = 2.363$ m

Que 1.22. **Define conveyance of a channel. Find the discharge in a trapezoidal channel with a bed width of 10 m, side slope 1 : 1 and depth of flow of 0.2 m under uniform flow condition. Bed slope is 1×10^{-4} and Manning's roughness coefficient 0.025. Also find Chezy's coefficient at this depth.**

AKTU 2014-15, Marks 10

Answer

A. Conveyance : We know that, $Q = \frac{1}{n} AR^{2/3}S_0^{1/2} \Rightarrow Q = k\sqrt{S_0}$

where, $k = \frac{1}{n} AR^{2/3}$ is called the conveyance of the channel and expresses the discharge capacity of the channel per unit longitudinal slope.

B. Numerical :

Given : Manning's coefficient, $n = 0.025$, Uniform flow depth, $y = 0.2$ m
Width of channel, $B = 10$ m, Side slope, $m = 1$, Bed slope, $S_0 = 0.0001$
To Find : Discharge, Chezy's coefficient.

1. Area, $A = (B + my)y = (10 + 1 \times 0.2) \times 0.2 = 2.04 \text{ m}^2$
2. Wetted perimeter, $P = B + 2y\sqrt{m^2+1} = 10 + 2 \times 0.2\sqrt{1^2+1}$
   $P = 10.57$ m
3. Hydraulic mean radius, $R = \frac{A}{P} = \frac{2.04}{10.57} = 0.193$ m
4. Mean velocity, $v = \frac{1}{n} R^{2/3} S_0^{1/2} = \frac{1}{0.025} \times (0.193)^{2/3} (0.0001)^{1/2}$
   $v = 0.134$ m/sec
5. Discharge, $Q = Av = 2.04 \times 0.134 = 0.273 \text{ m}^3/\text{sec}$
6. Chezy's coefficient, $C = \frac{1}{n} R^{1/6} = \frac{1}{0.025} \times (0.193)^{1/6} = 30.41$

Que 1.23. **A trapezoidal channel $B = 3.0$ m, $m = 1.50$, $n = 0.025$ and $S_0 = 0.0005$ takes off from a reservoir with free inlet. The reservoir elevation is 7.0 m above the channel bed at the inlet. Calculate the discharge in the channel by neglecting entrance losses.**

AKTU 2015-16, Marks 10

Answer

Given : Width of channel, $B = 3.0$ m, Side slope, $m = 1.50$
Manning's coefficient, $n = 0.025$, Bed slope, $S_0 = 0.0005$
Depth of flow, $y = 7.0$ m
To Find : Discharge in the channel.

1. Area of the channel, $A = (B + my)y = (3.0 + 1.50 \times 7.0) \times 7.0$
   $A = 94.5 \text{ m}^2$
2. Perimeter, $P = B + 2y(\sqrt{m^2+1}) = 3.0 + 2 \times 7.0 \times \sqrt{(1.50^2+1)}$
   $P = 28.24$ m
3. Hydraulic radius, $R = A/P = \frac{94.5}{28.24} = 3.35$ m
4. Manning's formula for discharge, $Q = \frac{1}{n} AR^{2/3}S_0^{1/2}$

$$Q = \frac{1}{0.025} \times 94.5 \times (3.35)^{2/3} \times (0.0005)^{1/2} = 189.24 \text{ m}^3/\text{sec.}$$

Que 1.24. **A channel is 2 m width at bottom, the length of each sloping side is 1.95 m. The width of water surface is 5.5 m. The flow depth is 1.2 m and bed slope is 1 in 5280. What is the discharge per minute ? Take value of $C = 34.6$.**

AKTU 2018-19, Marks 07

Answer

Given : Width of bottom, $B = 2$ m, Length of sloping side = 1.95 m, Top width of water surface, $T = 5.5$ m, Depth of flow, $y = 1.2$ m, Bed slope, $S = 1/5280$, Chezy's constant, $C = 34.6$
To Find : Discharge/min

1. Area of channel, $A = (5.5 + 2) \times 1.2 / 2 = 4.5 \text{ m}^2$
2. Wetted perimeter, $P = 2 \times \sqrt{(1.75^2 + 1.2^2)} + 2 = 6.244$ m
3. Hydraulic radius, $R = A/P = 4.5 / 6.244 = 0.7207$ m
4. Discharge is given by, $Q = AC\sqrt{RS}$

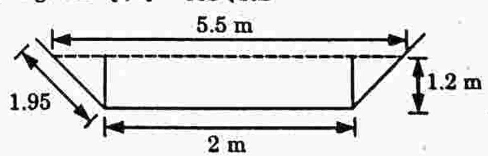


Fig. 1.24.1.

$$Q = 4.5 \times 34.6 \times \sqrt{0.7207 / 5280} = 1.819 \text{ m}^3/\text{sec} = 109.144 \text{ m}^3/\text{min}$$

Que 1.25. **On what factors does the Manning's rugosity coefficient depends ?** **AKTU 2014-15, Marks 05**

Answer

1. Following are the important factors which affects the Manning's rugosity coefficient :
   - i. Surface roughness.
   - ii. **Vegetation :** The presence of vegetation on the channel bed and sides leads to additional resistance and flexible vegetation, bending over of the vegetation tends to reduce the resistance and results in lower value of n.
   - iii. **Cross-Section Irregularity :** The channel irregularities and curvature, especially in natural streams, produce energy losses. As such, they are combined with the boundary resistance by suitably increasing the value of n. The sudden change in channel cross-section enhances the value of n.
   - iv. The presence of obstructions, e.g., bridge piers and transport of sediment load, either in suspension or near the bed, increases the value of n since it causes additional head loss.
   - v. Irregular alignment of channel.

Que 1.26. **A triangular channel with an apex angle of 75° carries a flow of 1.2 m³/sec at a depth of 0.80 m. If the bed slope is 0.009, find the roughness coefficient of the channel.** **AKTU 2015-16, Marks 05**

Answer

Given : Normal depth, $y_0 = 0.8$ m, Discharge, $Q = 1.2$ m³/sec
Bed slope, $S_0 = 0.009$, Apex angle, $\theta = 75°$
To Find : Roughness coefficient of the channel.

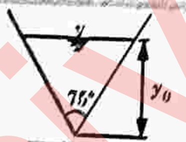


Fig. 1.26.1.

1. From Fig. 1.26.1, Area, $A = \frac{1}{2} \times 0.80 \times 2 \times 0.8 \times \tan\left(\frac{75°}{2}\right) = 0.491 \text{ m}^2$
2. Wetted perimeter, $P = 2 \times 0.8 \times \sec(37.5°) = 2.017$ m
3. Hydraulic radius, $R = \frac{A}{P} = \frac{0.491}{2.017} = 0.243$ m
4. Roughness coefficient, $n = \frac{AR^{2/3}S_0^{1/2}}{Q} = \frac{0.491 \times (0.243)^{2/3} \times (0.009)^{1/2}}{1.2}$
$= 0.015$

PART-4

Most Efficient Channel Section, Compound Sections.

CONCEPT OUTLINE

Most Efficient or Most Economical Channel Section : A section of channel is said to be economical when the cost of construction of the channel is minimum. Cost of construction depends upon the excavation and the lining.

Compound Channel : The term compound channel or two-stage channel, generally represents a channel with a central main channel flanked by flood plains on either side. The flow mostly takes place in the main channel but spills over to the floodplains at higher discharge.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.27. **What do understand by most efficient channel section ?**

Answer

1. Hydraulically most efficient section of a channel can be defined as the one which passes maximum discharge for given area, roughness and bed slope.
2. Discharge can be written as, $Q = A \times v = A \times (1/n) R^{2/3} S^{1/2}$
3. Hence for maximum Q, for given A, n and S the hydraulic radius R should be maximum, *i.e.*, $R = (A/P)$ is maximum. It means that perimeter P should be minimum.
4. Therefore, the most efficient section can also be said as the one which has minimum wetted perimeter P.

Que 1.28. **Show that for a rectangular channel with given area is most efficient when hydraulic radius is half of the depth of the flow.**

AKTU 2014-15, Marks 05

OR

State the conditions under which the rectangular section of an open channel will be most economical. Derive these conditions.

AKTU 2016-17, Marks 10

Answer

1. Let, bottom width = B and Depth of flow = y
2. Area of flow, $A = By$ (Constant)
3. Wetted perimeter, $P = B + 2y = (A/y) + 2y$
4. If P is to be minimum with constant A, then $dP/dy = 0$

$$-\frac{A}{y^2} + 2 = 0 \Rightarrow A = 2y^2$$

$$By = 2y^2 \Rightarrow y = B/2$$

5. Hydraulic mean radius, $R = \dfrac{A}{P} = \dfrac{2y^2}{4y} = \dfrac{y}{2}$

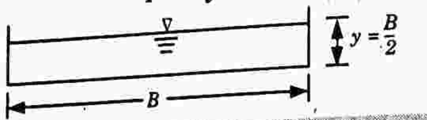


Fig. 1.28.1. Hydraulically efficient rectangular channel.

6. Thus, for a rectangular channel when the depth of flow is equal to half the bottom width, *i.e.*, when the channel section is a half-square, a hydraulically efficient section is obtained.

Que 1.29. **Show that a hydraulically efficient triangular channel section had $R_e = \dfrac{y_e}{2\sqrt{2}}$. All symbols have usual meanings.**

Answer

1. For a triangular section with semi-apex angle θ,

   Area of channel, $A = (1/2) \times 2\,my_e \times y_e = my_e^2 \Rightarrow y_e = \sqrt{A/m}$

2. Wetted perimeter, $P = 2y_e\sqrt{m^2+1} = 2\sqrt{A/m} = \sqrt{m^2+1}$

$$P^2 = 4(A/m)(m^2+1)$$

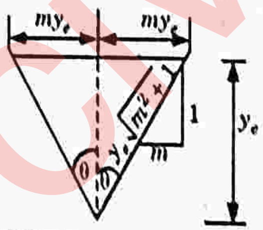


Fig. 1.29.1. Hydraulically efficient triangular section.

3. For most efficient section, $dP/dm = 0$

$$2P\frac{dP}{dm} = 4A\left(1 - \frac{1}{m^2}\right) = 0 \;\Rightarrow\; m^2 - 1 = 0 \;\Rightarrow\; m = 1$$

4. Hence apex angle, $2\theta = 90°$

∴ The most efficient triangular section is half of a square with diagonal horizontal.

5. Hydraulic radius, $R_e = \dfrac{A}{P} = \dfrac{my_e^2}{2y_e\sqrt{m^2+1}} = \dfrac{1 \times y_e^2}{2y_e\sqrt{1^2+1}} = \dfrac{y_e}{2\sqrt{2}}$

Que 1.30. **Show that for a trapezoidal channel of given area of flow. The condition of maximum flow requires that hydraulic mean depth is equal to one-half the depth of flow.**

AKTU 2016-17, Marks 10

OR

Derive the condition for most efficient trapezoidal channel section for uniform flow.

AKTU 2018-19, Marks 07

OR

Prove that hydraulically most efficient trapezoidal section is half of regular hexagon.

AKTU 2017-18, Marks 07

Answer

1. Let, Bottom width = B and Side slope = $m : 1$ = (H : V)
2. Area, $A = (B + my)y$ = Constant ...(1.30.1)

$$B = \frac{A}{y} - my \quad ...(1.30.2)$$

3. Wetted perimeter, $P = B + 2y\sqrt{m^2+1}$

$$= \frac{A}{y} - my + 2y\sqrt{m^2+1} \quad ...(1.30.3)$$

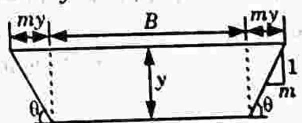


Fig. 1.30.1. Hydraulically efficient trapezoidal channel.

4. For hydraulically efficient section, keeping m and A constant.

$$\frac{dP}{dy} = 0 \Rightarrow -\frac{A}{y^2} - m + 2\sqrt{m^2+1} = 0$$

$$A = (2\sqrt{m^2+1} - m)y^2 \quad ...(1.30.4)$$

5. From eq. (1.30.2), we get

$$B = \frac{A}{y} - my = 2(\sqrt{m^2+1} - m)y \quad ...(1.30.5)$$

6. From eq. (1.30.3), we get

$$P = B + 2y\sqrt{m^2+1} = 2y(2\sqrt{m^2+1} - m) \quad ...(1.30.6)$$

7. Hydraulic mean radius, $R = \frac{A}{P} = \frac{(2\sqrt{1+m^2}-m)y^2}{2(2\sqrt{1+m^2}-m)y} = \frac{y}{2}$

8. For most efficient channel,

$$\frac{dP}{dm} = 0 \Rightarrow 2y\left[2 \times \frac{1}{2}(1+m^2)^{-1/2} \times 2m - 1\right] = 0$$

∴

$$\frac{2m}{\sqrt{1+m^2}} - 1 = 0 \Rightarrow m = \frac{1}{\sqrt{3}} \qquad [\because y \neq 0]$$

$$1/\tan\theta = 1/\sqrt{3} \Rightarrow \tan\theta = \sqrt{3} = \tan 60° \Rightarrow \theta = 60°$$

9. Put the value of m in eq. (1.30.4), eq. (1.30.5) and eq. (1.30.6), we get

$$A = \left(2\sqrt{1+\frac{1}{3}} - \frac{1}{\sqrt{3}}\right)y^2 = \sqrt{3}y^2$$

$$B = 2y\left(\sqrt{1+\frac{1}{3}} - \frac{1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}y$$

∴

$$P = 2y\left(2\sqrt{1+\frac{1}{3}} - \frac{1}{\sqrt{3}}\right) = 2\sqrt{3}y$$

10. If L = Length of the inclined side of the channel, then

$$L = \frac{P-B}{2} = \frac{2}{\sqrt{3}}y = B$$

11. Hence the conditions for the most economical trapezoidal section are :

i. Hydraulic mean radius, $R = y/2$

ii. Angle of the inclined sides of the channel from the bed, $\theta = 60°$.

iii. Thus the hydraulically most efficient trapezoidal section is one half of a regular hexagon.

Que 1.31. **Show that for maximum velocity through a circular channel, the depth of flow and hydraulic radius are given by,**

i. $y = 0.81\,D$ **ii. $R = 0.3055\,D$**

Answer

Condition for Maximum Velocity :

1. Consider liquid is flowing through a circular channel of radius r as shown in Fig. 1.31.1. Let the depth of flow of liquid be y and the liquid surface subtend an angle 2θ with the axis of the channel.

2. The cross-section area of flow for a circular channel is given by,

$$A = r^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

3. Wetted perimeter for a circular channel is given by, $P = 2r\theta$

Thus,

$$R = \frac{A}{P} = \frac{r^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2r\theta} = \frac{r}{2} - \frac{r\sin 2\theta}{4\theta}$$

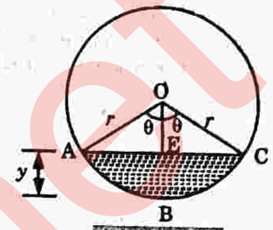


Fig. 1.31.1.

4. Using Chezy's equation, the velocity of flow through a channel can be expressed as

$$v = C\sqrt{RS} = C\sqrt{\frac{A}{P}S}$$

5. For a circular channel, with constant values of Chezy's constant C and the slope of the bed of the channel S, the only variable is q and the condition for the maximum velocity is

$$\frac{d(A/P)}{d\theta} = 0 \Rightarrow d\left(\frac{r}{2} - \frac{r\sin 2\theta}{4\theta}\right) = 0$$

$$\frac{d\left(\frac{r\sin 2\theta}{4\theta}\right)}{d\theta} = 0 \Rightarrow \frac{r \times 2\theta \times \cos 2\theta - r\sin 2\theta}{\theta^2} = 0$$

After solving we get, $\theta = 128°45'$

6. Depth of flow is then given by (Refer Fig. 1.31.1)

$$y = r - r\cos\theta = r(1 + 0.62) = 1.62r = 0.81D \qquad ...(1.31.1)$$

7. Eq. (1.31.1) implies that for maximum velocity of flow, the depth of flow in the circular channel will be equal to 0.81 times the diameter of the channel.

8. Hydraulic radius is then found to be

$$R = \frac{r}{2} - \frac{r\sin 2\theta}{4\theta} = \frac{r}{2} - \frac{r\sin 257°30'}{4(2.247)} = 0.611r = 0.3055D \qquad ...(1.31.2)$$

9. Eq. (1.31.2) implies that for maximum velocity of flow through a circular channel, the hydraulic radius will be equal to 0.3055 times the diameter of the channel.

Que 1.32. **Show that for maximum discharge of flow through a circular channel the depth of flow and hydraulic radius are given by,**

i. $y = 0.95\,D$ **ii. $R = 0.266\,D$**

Answer

Condition for Maximum Discharge :

1. The flow area for a circular channel is given by, $A = r^2\left(\theta - \frac{\sin 2\theta}{2}\right)$

Therefore, $dA/d\theta = r^2(1 - \cos 2\theta)$

2. Wetted perimeter for a circular channel is given by, $P = 2r\theta$
3. Using Chezy's equation the discharge through a channel can be expressed as

$$Q = CA\sqrt{RS} = CA\sqrt{\frac{A}{P}S} = C\sqrt{\frac{A^3}{P}S}$$

4. Therefore, for constant values of Chezy's constant C and the slope of the bed of the channel S, the condition for the maximum discharge is

$$\frac{d\,(A^3/P)}{d\theta} = 0 \Rightarrow \frac{P \times 3A^2 \times \frac{dA}{d\theta} - A^3\frac{dP}{d\theta}}{d\theta} = 0$$

$$3P\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0$$

5. Substituting the respective values, we get

$$3 \times 2r\theta \times r^2(1 - \cos 2\theta) - r^2\left(\theta - \frac{\sin 2\theta}{2}\right)2r = 0$$

$$3\theta(1 - \cos 2\theta) - \left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

$$4\theta - 6\theta \cos 2\theta - + \sin 2\theta = 0 \quad \text{...(1.32.1)}$$

The solution of eq. (1.32.1) gives, $\theta = 154°(2.6878 \text{ rad})$

6. Depth of flow is then given by, $y = r - r\cos\theta$

or $y = r(1 + 0.9) = 1.9r = 0.95\,D$ ...(1.32.2)

7. Eq. (1.32.2) implies that for maximum discharge, the depth of liquid in the circular channel will be equal to 0.95 times the diameter of the channel.
8. Hydraulic radius is then computed as

$$R = \frac{r}{2} - \frac{r\sin 2\theta}{4\theta} = \frac{r}{2} - \frac{r\sin 308°}{4(2.6878)} = 0.5732r = 0.2866D \quad \text{...(1.32.3)}$$

9. Eq. (1.32.3) implies that for maximum discharge through a circular channel, the hydraulic radius will be equal to 0.2866 times the diameter of the channel.

Que 1.33. **An open channel to be made of concrete is to be designed to carry 1.5 m³/s at a slope of 0.00085. Find the most efficient cross section for (a) Rectangular section (b) Trapezoidal section (c) Semicircular section.**

AKTU 2017-18, Marks 07

Answer

Given : Discharge, $Q = 1.5$ m³/sec, Slope, $S = 0.00085$

To Find : Most efficient cross section for

a. Rectangular section,
b. Trapezoidal section,
c. Semicircular section.

A. Rectangular Section :

Assume Chezy's constant, $C = 50$

1. For the rectangular channel to be most economical,

i. Width, $b = 2d$

ii. Hydraulic mean depth, $R = d/2$

2. Area of flow, $A = bd = 2d \times d = 2d^2$
3. Discharge, $Q = AC\sqrt{RS} \Rightarrow 1.5 = 2d^2 \times 50\sqrt{\frac{d}{2} \times 0.00085}$

After solving, we get

Depth of channel, $d = 0.88$ m

4. Width of channel, $b = 2d = 2 \times 0.88 = 1.76$ m

B. Trapezoidal Section :

1. Let trapezoidal channel has side slope 1H : 2V, for most economical section.

$$\frac{b + 2nd}{2} = d\sqrt{n^2 + 1} \Rightarrow \frac{b + 2 \times \frac{1}{2} \times d}{2} = d\sqrt{\left(\frac{1}{2}\right)^2 + 1}$$

$$\frac{b + d}{2} = d\sqrt{\frac{5}{4}} = 1.118\,d \Rightarrow b = 1.236\,d \quad \text{...(1.33.1)}$$

2. But area of trapezoidal section,

$$A = \frac{b + (b + 2nd)}{2} \times d = (b + nd)\,d \quad \text{...(1.33.2)}$$

From eq. (1.33.1) and eq. (1.33.2), we get

$$A = \left(1.236d + \frac{d}{2}\right)d \Rightarrow A = 1.736\,d^2$$

3. Discharge, $Q = AC\sqrt{RS}$ $\left(\because R = \frac{d}{2}\right)$

$$1.5 = 1.736\,d^2 \times 50 \times \sqrt{\frac{d}{2} \times 0.00085}$$

4. Depth of section, $d = 0.932$ m
5. Bottom width of section, $b = 1.236 \times d = 1.236 \times 0.932 = 1.152$ m

C. Circular Section :

1. For maximum velocity, $\theta = 128°\,45'$ or 2.247 Radians.
2. Wetted perimeter, $P = 2r\,\theta$.
3. Area of flow, $A = r^2\left(\frac{\theta - \sin 2\theta}{2}\right)$
4. Discharge, $Q = AC\sqrt{RS}$

$$1.5 = r^2\left(2.247 - \frac{\sin 2\times(128°\,45')}{2}\right)\times 50\times\sqrt{\frac{A}{P}\times 0.00085}$$

$$1.5 = r^2\times 2.735\times 50\times\sqrt{\frac{r^2\times 2.735}{2\times r\times 2.247}\times 0.00085}$$

Radius, $r = 0.747$ m

Diameter, $D = 1.494$ m

Que 1.34. **A rectangular channel carries water at the rate of 400 litres/sec when bed slope is 1 in 2000. Find the economical dimensions of the channel if $C = 50$.** **AKTU 2018-19, Marks 07**

Answer

Given : Discharge, Q = 400 litres/sec, Bed slope, S = 1/2000, Chezy's constant, $C = 50$

To Find : Dimension of most economical rectangular channel section.

1. Let width of channel = B, Depth of channel = y
2. Most economical dimensions for rectangular channel,

$$R = y/2 \text{ and } B = 2y$$

3. Discharge is given by, $Q = AC\sqrt{RS} = ByC\sqrt{RS}$

$$Q = 2y^2 C\sqrt{\frac{y}{2}\times S}$$

$$\frac{400}{1000} = 2y^2\times 50\sqrt{\frac{y}{2}\times\frac{1}{2000}}$$

$$y = 0.577 = 0.58 \text{ m}$$

Depth, $y = 0.58$ m

Width, $B = 2y = 2\times 0.58 = 1.16$ m

Que 1.35. **A concrete lined circular channel of 3 m diameter has bed slope of 1 in 5000. Determine the velocity and the flow rate for the condition of :**

i. Maximum velocity.

ii. Maximum discharge.

Take Chezy's constant, $C = 50$.

Answer

Given : Diameter of circular channel, $D = 3$ m

Bed slope, $S = \frac{1}{5000}$, Chezy's constant, $C = 50$

To Find : Velocity and flow rate.

Let 2θ be the total angle subtended by the water surface at the centre of the channel.

A. Maximum Velocity Condition :

1. For maximum velocity total angle subtended,

$$2\theta = 257.5° = 257.5\times\frac{\pi}{180} = 4.49 \text{ rad}$$

2. Depth of flow, $y = 0.81\,D = 0.81\times 3 = 2.43$ m
3. Area of flow $A = \frac{r^2}{2}(2\theta - \sin 2\theta) = \frac{1.5^2}{2}(4.49 - \sin 257.5°)$

$A = 6.15 \text{ m}^2$

4. Wetted perimeter, $P = 2r\theta = r\times 2\theta = 1.5\times 4.49 = 6.735$ m
5. Hydraulic radius, $R = \frac{A}{P} = \frac{6.15}{6.735} = 0.913$ m
6. Flow velocity, $v = C\sqrt{RS} = 50\sqrt{0.913\times\frac{1}{5000}} = 0.675$ m/sec
7. Flow rate, $Q = Av = 6.15\times 0.675 = 4.15 \text{ m}^3/\text{sec}$

B. Maximum Discharge Condition :

1. For maximum discharge total angle,

$$2\theta = 308° = 308\times\frac{\pi}{180} = 5.375 \text{ rad}$$

2. Depth of flow, $y \doteq 0.95\,D = 0.95\times 3 = 2.85$
3. Area of flow, $A = \frac{r^2}{2}(2\theta - \sin 2\theta)$

$$A = \frac{1.5^2}{2}(5.375 - \sin 308°) = 6.93 \text{ m}^2$$

4. Wetted perimeter, $P = 2r\theta = r\times 2\theta = 1.5\times 5.375 = 8.063$ m
5. Hydraulic radius, $R = \frac{A}{P} = \frac{6.93}{8.063} = 0.86$ m
6. Flow velocity, $v = C\sqrt{RS} = 50\sqrt{0.86\times\frac{1}{5000}} = 0.66$ m/sec
7. Flow rate, $Q = Av = 6.93\times 0.66 = 4.57 \text{ m}^3/\text{sec}$

Que 1.36. **What is compound channel ? How would you calculate the total discharge of compound channel ? Explain with example.**

Answer

A. Compound Channel :

1. A compound channel is a channel section composed of a main deep portion and one or two flood plains that carry high-water flows.
2. The main channel carries the dry weather flow and during wet season, the flow may spillover the banks of the main channel to the adjacent flood plains.

3. A majority of natural rivers have compound sections. A compound section is also known as two-stage channel.

B. Total Discharge Calculation Methods :

The following two methods are popular, and give reasonably good results :

1. Vertical Interface Method :

i. In this method the flood banks are separated from the main channel by means of vertical interface as shown in Fig. 1.36.1. This interface is considered as a surface of zero shear where is no transfer of momentum takes place.

ii. The length of the vertical interface is not included in the calculation of the wetted perimeter of either the over bank flow or the main channel flow.

2. Diagonal Interface Method :

i. In this method, a diagonal interface is considered from the top of the main channel bank to the centerline of the water surface as shown in Fig. 1.36.1.

ii. This interface is considered to be a surface of zero shear stress and as such the length of the diagonal interfaces are not included in the calculation of the wetted perimeters of the over bank and main channel flows.

iii. If the flow over bank portion has significant roughness discontinuities equivalent roughness for over bank region can be adopted.

C. Example : A compound channel is symmetrical in cross section and has the following geometric properties.

Main Channel : Trapezoidal cross section, bottom width = 15.0 m. Side slopes = 1.5 H : 1, bank flood depth = 3.0 m, Manning's coefficient = 0.03, longitudinal slope = 0.0009.

Flood Plains : Width = 75 m, Side slope = 1.5 H : 1 V, Manning's coefficient = 0.05, Longitudinal slope = 0.0009. Compute the uniform flow discharge for a flow with total depth of 4.2 m.

Vertical Interface Procedure :

1. The channel section is considered divided into three subsection, A_1, A_2 and A_3 by means of two vertical interfaces which start at the intersection of the flood plains and the main channel as shown in Fig. 1.36.1.

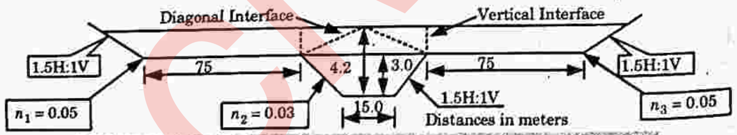


Fig. 1.36.1. Schematic representation of compound channel.

2. The calculation of area, wetted perimeters of each of the three sub-areas is given in table 1.36.1.

By symmetry, sub-area A_1 = sub-area A_3. The vertical interfaces are as indicated in Fig. 1.36.1.

3. As per the rules of this computation procedure, the interfaces are treated as surfaces of zero shear stress and hence are not included in the calculation of the wetted perimeter.

Table 1.36.1. Computation of geometrical properties of vertical interfaces.

Sub-area	Area Element	Area (m²)		Wetted Perimeter (m)		Hyd. Radius (m)
A_1	A_{11}	[0.5×1.2×(1.5×1.2)]	1.08	$1.2\times(1+1.5^2)^{0.5}$	2.163	
	A_{12}	75×1.2	90	75	75	
	Total		91.08		77.163	1.180
A_2	A_{21}	[15+(1.5×3.0)]×3.0	58.5	$15+2\times3.0\times[1+(1.5)^2]^{0.5}$	25.817	
	A_{22}	[15+(2×1.5×3)]×1.2	28.8	0	0	
	Total		87.3		25.817	3.382

4. Discharge by Manning's formula :

i. For sub-area A_1, $Q_1 = \dfrac{1}{0.05} \times 91.08 \times (1.180)^{2/3} \times (0.0009)^{1/2} = 61.035 \text{ m}^3/\text{sec}$

ii. For sub-area A_2, $Q_2 = \dfrac{1}{0.03} \times 87.3 \times (3.382)^{2/3} \times (0.0009)^{1/2} = 196.697 \text{ m}^3/\text{sec}$

iii. For sub-area A_3, $Q_2 = \dfrac{1}{0.05} \times 91.08 \times (1.180)^{2/3} \times (0.0009)^{1/2} = 61.03 \text{ m}^3/\text{sec}$

5. Total discharge, $Q = Q_1 + Q_2 + Q_3 = 318.77 \text{ m}^3/\text{sec}$

Que 1.37. **A compound channel is symmetrical in cross section and has the following geometric properties. Main channel : Trapezoidal cross section, Bottom width = 15.0 m, Side slopes = 1.5 H : 1 V, Bank full depth = 3.0 m, Manning's coefficient = 0.03, Longitudinal slope = 0.0009, Flood plains : Width = 75 m, Side slope = 1.5 H : 1 V, Manning's coefficient = 0.05, Longitudinal slope = 0.0009. Compute the uniform flow discharge for a flow with total depth of 4.2 m by using DCM with either (*i*) diagonal interface, or (*ii*) vertical interface procedures.**

AKTU 2015-16, Marks 12

Answer

A. Diagonal Interface Procedure :

1. The channel section is considered divided into three subsections A_1, A_2 and A_3, by means of two diagonal interfaces as shown in Fig. 1.37.1.

2. The calculation of area and wetted perimeters of each of the three sub-areas is given in table 1.37.1.

By symmetry, Sub-area A_1 = Sub-area A_3. The diagonal interfaces are shown in Fig. 1.37.1.

3. As per the rules of this procedure, the interfaces are treated as surfaces of zero shear stress and hence are not included in the calculation of the wetted perimeter.

Table 1.37.1. Computation of geometrical properties of compound channel.

Sub-area	Area Element	Area (m^2)		Wetted Perimeter(m)		Hyd. Radius (m)
$A_1 = A_3$	A_{11}	$[0.5 \times 1.2 \times (1.5 \times 1.2)]$	1.08	$1.2 \times [1 + (1.5)^{\times 2}]^{0.5}$	2.163	
	A_{12}	75×1.2	90	75	75	
	A_{13}	$[(0.5 \times 15) + (1.5 \times 3)] \times 0.5 \times 1.2$	7.2	0	0	
	Total		98.28		77.16	1.274
A_2	A_{21}	$[15 + (1.5 \times 3.0)] \times 3.0$	58.5	$15 + 2 \times 3.0 \times [1 + (1.5)^2]^{0.5}$	25.82	
	A_{22}	$[15 + (2 \times 1.5 \times 3)] \times 0.5 \times 1.2$	14.4	0	0	
	Total		72.9		25.82	2.824

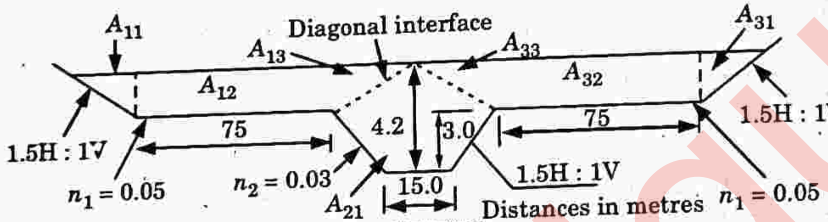


Fig. 1.37.1.

5. **Discharge by Manning's Formula :**

i. For sub-area A_1, $Q_1 = \dfrac{1}{0.05} \times 98.28 \times (1.274)^{2/3} \times (0.0009)^{1/2} = 69.3 \text{ m}^3/\text{sec}$

ii. For sub-area

$$A_2, Q_2 = (1/0.03) \times 72.90 \times (2.824)^{2/3} \times (0.0009)^{1/2} = 145.650 \text{ m}^3/\text{sec}$$

iii. For sub-area A_3,

$$Q_3 = Q_1 = \frac{1}{0.05} \times 98.28 \times (1.274)^{2/3} \times (0.0009)^{1/2} = 69.3 \text{ m}^3/\text{sec}$$

6. Total discharge,

$$Q = Q_1 + Q_2 + Q_3 = 69.3 + 145.650 + 69.3 = 284.25 \text{ m}^3/\text{sec}$$

B. Vertical Interface Procedure : Refer Q. 1.36, Page 1–32B, Unit-1.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Differentiate between pipe flow and open channel flow.

Ans. Refer Q. 1.1, Unit-1.

Q. 2. Draw typical velocity profile at a vertical section of an open channel.

Ans. Refer Q. 1.4, Unit-1.

Q. 3. What do you understand by specific energy for a flow in open channel ? Draw the specific energy diagram and describe its various characteristics.

Ans. Refer Q. 1.7, Unit-1.

Q. 4. What is critical depth in open-channel flow ? For a given average flow velocity, how is it determined ?

Ans. Refer Q. 1.10 and Q. 1.11; Unit-1.

Q. 5. Show that in rectangular channel maximum discharge occurs when the flow is critical for a given value of specific energy.

Ans. Refer Q. 1.14, Unit-1.

Q. 6. A rectangular channel 2.5 m wide has a specific energy of 1.50 m when carrying a discharge of 6.48 m³/sec. Calculate the alternate depths and corresponding Froude numbers.

Ans. Refer Q. 1.15, Unit-1.

Q. 7. What is Chezy's formula ? How is it derived ?

Ans. Refer Q. 1.17, Unit-1.

Q. 8. Define conveyance of a channel. Find the discharge in a trapezoidal channel with a bed width of 10 m, side slope 1 : 1 and depth of flow of 0.2 m under uniform flow condition. Bed slope is 1×10^{-4} and Manning's roughness coefficient 0.025. Also find Chezy's coefficient at this depth.

Ans. Refer Q. 1.22, Unit-1.

Q. 9. State the conditions under which the rectangular section of an open channel will be most economical. Derive these conditions.

Ans. Refer Q. 1.28, Unit-1.

Q. 10. Derive the condition for most efficient trapezoidal channel section for uniform flow.

Ans. Refer Q. 1.30, Unit-1.

Q. 11. What is compound channel ? How would you calculate the total discharge of compound channel ? Explain with example.

Ans. Refer Q. 1.36, Unit-1.

☺☺☺

2

UNIT

Uniform Flow in Open Channel

CONTENTS

PART-1

Energy-Depth Relationship : Application of Specific Energy Principle for Interpretation of Open Channel Phenomena, Flow Through Vertical and Horizontal Contractions.

CONCEPT OUTLINE

Application of Specific Energy Principle for Interpretation of Open Channel Phenomenon : Many times it becomes necessary to provide transition for a very long channel section.

A transition is the portion of a channel with varying cross-section used for connecting one uniform channel to another. Both the channels may or may not have same cross-section.

Transition may be Two Types :

1. **Sudden Transition :** In this type the change in cross-section of channel section occurs in relatively shorter length.
2. **Gradual Transition :** In this type the change in cross-section of channel section occurs in relatively longer length.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. **Explain the application of specific energy principle for the interpretation of open channel flow through**

A. Horizontal contractions.

B. Vertical contractions.

Answer

A. ***Horizontal Contractions (Transition with Reduction in Width) :***

1. Consider a rectangular channel section. The width of the channel is reduced as shown in the Fig. 2.1.1.
2. The portion of the channel section with reduced width is known as throat section of simple throat.
3. The following assumption are considered :
   i. No loss of energy takes place between the two sections.
   ii. The bottom of the channel is horizontal.
   iii. Specific energy is same at both the sections under consideration.
4. Let total discharge = Q
   i. The width of channel at section (1)-(1) = B_1

ii. The width of channel at section (2)-(2) = B_2

Here $B_1 > B_2$

iii. $q_1 < q_2$ $\left(\because\ q_1 = \dfrac{Q}{B_1} \text{ and } q_2 = \dfrac{Q}{B_2}\right)$

and if $B_3 < B_2 < B_1$

$\therefore$ $q_3 > q_2 > q_1$

iv. Thus we can see that as the width of the channel is reduced the discharge per unit width will be increased.

v. We known $E = h + \dfrac{q^2}{2gh^2} \Rightarrow q = \sqrt{2g(E-h)h^2}$

5. **Characteristic of Flow Under Subcritical Condition :**

i. We can observe from the discharge diagram that in the subcritical region for a given specific energy 'E' as 'q' increases 'h' will decrease.

ii. This decrease in depth 'h' will continue till it reaches a minimum value known as critical depth 'h_c'.

iii. At this critical depth 'h_c' discharge per unit width will be maximum and denoted as 'q_{max}'.

iv. If we want to increase the discharge per unit width beyond the maximum value 'q_{max}' obtained at 'h_c' by farther decreasing the width then there will be a change in specific energy.

v. When we reduce the width for increasing the 'q' beyond 'q_{max}' then there will be rise in the free surface on the upstream side of section (2)-(2).

vi. The result in farther increase in specific energy to such extent that it can support the increased discharge through the contracted channel section.

vii. Hence for certain specific energy 'E' there will be a maximum value of q at a depth of h_c for particular value of channel with B and if B is decreased farther, the flow will attain a greater depth of flow than h_c which is practically not possible.

6. **Characteristics of Flow Under Super-critical Condition :**

i. It can also be seen from the discharge diagram that in super-critical region for a given value of 'E' as 'q' increases the value of 'h' will also increase.

ii. This increase in depth 'h' will continue till it attains a maximum value of depth of flow known as critical depth 'h_c'.

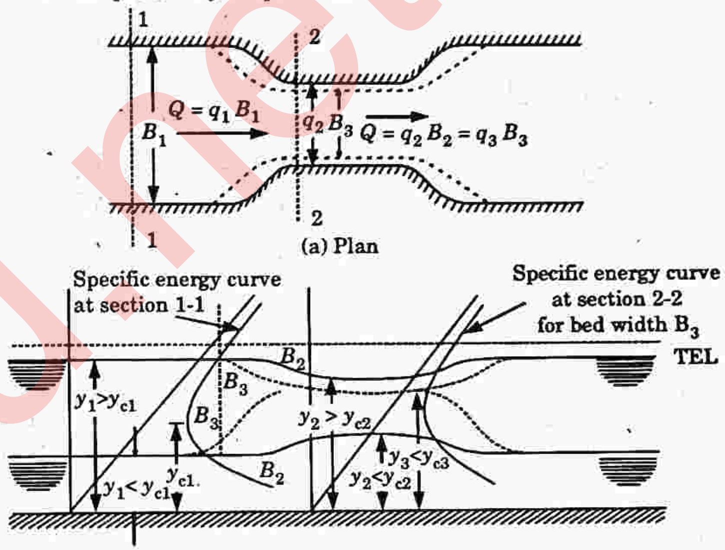
iii. At this maximum value of 'h' the discharge per unit width will also be maximum and denoted as 'q_{max}'.

iv. If we want to increase the discharge beyond the 'q_{max}' by farther decreasing the width then there will be change in specific energy.

v. When we reduce the width such that 'q' increases beyond 'q_{max}' the free surface on the upstream side of section (2)-(2) will be lowered.

vi. This lower level of free surface on the upstream side of section (2)-(2) will cause an increase in specific energy to such an extent that can support the increase discharge through contracted channel section.

vii. Hence that for a certain specific energy 'E' there will be maximum value of 'q' at a depth of 'h_c' for a particular value of channel width 'B'. If B is decreased farther then there will be an increased value of 'h' beyond 'h_c' which is practically not possible.



y_{c1} = Critical depth for bed width B_1

y_{c2} = Critical depth for bed width B_2

y_{c3} = Critical depth for bed width B_3

Fig. 2.1.1.

B. **Vertical Contractions (Transition with Raised Bottom in a Rectangular Channel) :**

1. Fig. 2.1.2 shows a rectangular channel section with uniform width and having a rise in bottom generally known as hump.
2. The width of the channel is constant thus the discharge per unit width 'q' will also be constant for a constant discharge 'Q'.
3. The specific energy 'E' is measured with respect to the channel bottom. It is assumed that there is no loss energy between section (1)-(1) and (2)-(2).
4. Due to rise in channel bottom there will be a decrease in specific energy.

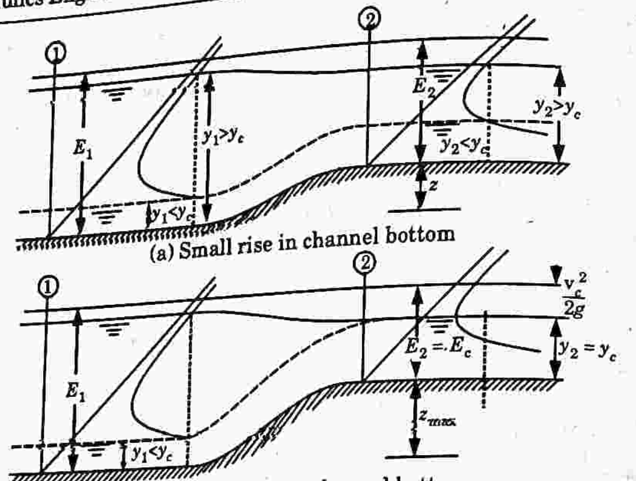
$$E_2 = E_1 - \Delta z$$

where, E_1 and E_2 = Specific energy at section (1)-(1) and section (2)-(2).

Δz = Height of the hump.

5. **Case 1 :** Flow approaching hump is in subcritical state.

i. The free surface drops down at the hump.



(a) Small rise in channel bottom

(b) Critical rise in channel bottom

Fig. 2.1.2. Flow in a rectangular channel with a hump in the bottom.

ii. Decrease in specific energy is associated with a decrease in depth of flow and increase in the velocity. Over the hump, the depth of flow decreases and the velocity head increase.

6. **Case 2 :** Flow approaching hump is in supercritical state.

i. The depth of flow at hump is increased.

ii. In case of supercritical flow a decrease in specific energy is associated with increase in depth of flow and decrease in velocity.

iii. Hence there is a limit upto which the specific energy for a given discharge can be reduced by increasing the height of hump. This maximum height of hump is denoted as Δz_{max}

$E_1 = E_c + \Delta z_{max}$, where, E_c = Specific energy at critical flow state.

iv. At this state the depth of flow will be equal to critical depth 'h_c' and velocity of flow will be equal to critical velocity v_c.

v. However, if the height of the hump is increased further, then in order to pass the same discharge the specific energy will have to be increased.

vi. In this case for subcritical flow approaching the hump the required increase in specific energy will be provided by increase in the depth of flow of the section (1)-(1) and for supercritical flow approaching the hump, the depth of flow at section (1)-(1) will be reduced thereby providing the required increase in specific energy.

Que 2.2. **A wide rectangular channel carries a flow of 2.75 m³/sec per meter width, the depth of flow being 1.5 m. Calculate the rise of the floor level required to produce a critical flow condition. What is the corresponding fall in surface level ?** **AKTU 2014-15, Marks 10**

Answer

Given : Discharge, q = 2.75 m³/sec/m, Depth, y = 1.5 m
To Find : Rise of the floor level required to produce a critical flow condition, Corresponding fall in surface level.

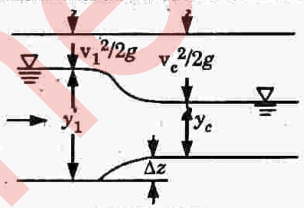


Fig. 2.2.1.

1. **Rise of Floor Level :**

i. Critical depth, $y_c = (q^2/g)^{1/3} = [(2.75)^2/9.81]^{1/3} = 0.917$ m

ii. We know that, $v_c^2/2g = y_c/2 = 0.459$

$y_1 = 1.5$ m, $v_1 = q/y_1 = 2.75/1.5 = 1.83$ m/sec

$v_1^2/2g = 1.83^2/(2 \times 9.81) = 0.171$ m

iii. By energy equation, $y_1 + (v_1^2/2g) = \Delta z + y_c + (v_c^2/2g)$

$1.5 + 0.171 = \Delta z + 0.917 + 0.459$

$\Delta z = 0.295$ m

2. **Fall in the Water Surface Elevation :**

$\Delta h = (v_c^2/2g) - (v_1^2/2g) = 0.459 - 0.171 = 0.288$ m

Que 2.3. **The width of a horizontal rectangular channel is reduced from 3.5 m to 2.5 m and the floor is raised by 0.25 m in elevation at a given section. At the upstream section, the depth of flow is 2.0 m and the kinetic energy correction factor α is 1.15. If the drop in the water surface elevation at the contraction is 0.20 m, calculate the discharge if (a) The energy loss is neglected, and (b) the energy loss is one-tenth of the upstream velocity head. [The kinetic energy correction factor at the contracted section may be assumed to be unity]** **AKTU 2015-16, Marks 10**

Answer

Given : Depth of flow, y_1 = 2 m, Width at upstream, B_1 = 3.5 m
Floor level raised at downstream = 0.25 m,
Width at downstream, B_2 = 2.5 m.
Kinetic energy correction factor α_1 = 1.15 and α_2 = 1, Drop in the water surface elevation at downstream = 0.20 m
To Find : Discharge.

1. Downstream depth of flow, $y_2 = 2.0 - 0.25 - 0.20 = 1.55$ m

2. By continuity equation, $B_1 y_1 v_1 = B_2 y_2 v_2$

$$v_1 = \frac{2.5 \times 1.55}{3.5 \times 2.0} \times v_2 = 0.5536\, v_2$$

3. **When there is No Energy Loss :**

i. By energy equation applied to section 1 and 2,

$$z_1 + y_1 + \alpha_1 (v_1^2/2g) = (z_1 + \Delta z) + y_2 + \alpha_2 (v_2^2/2g)$$

$$\frac{v_2^2}{2g} - \frac{1.15 \times (0.5536)^2\, v_2^2}{2g} = 2 - 1.55 - 0.25 \;\Rightarrow\; v_2 = 2.462 \text{ m/sec}$$

ii. Discharge, $Q = 2.5 \times 1.55 \times 2.462 = 9.54 \text{ m}^3/\text{sec}$

4. **When there is an Energy Loss :**

$$H_L = 0.1[\alpha_1(v_1^2/2g)] = 0.115\,(v_1^2/2g)$$

i. By energy equation, $z_1 + y_1 + \alpha_1 \dfrac{v_1^2}{2g} = (z_1 + \Delta z) + y_2 + \alpha_2 \dfrac{v_2^2}{2g} + H_L$

$$\alpha_2(v_2^2/2g) - \alpha_1 (v_1^2/2g) + H_L = y_1 - y_2 - \Delta z \quad ...(2.3.1)$$

ii. Substituting the value of $\alpha_2 = 1.0$, $\alpha_1 = 1.15$ and $H_L = 0.115\,(v_1^2/2g)$ in eq. (2.3.1)

$$\frac{v_2^2}{2g} - 1.15\frac{v_1^2}{2g} + 0.115\frac{v_1^2}{2g} = 2.00 - 1.55 - 0.25 \quad [\because v_1 = 0.5536\, v_2]$$

$$(v_2^2/2 \times 9.81)\,[1 - 1.15 \times (0.5536)^2 + 0.115 \times (0.5536)^2] = 0.2$$

$$v_2 = 2.397 \text{ m/sec}$$

iii. Discharge, $Q = 2.5 \times 1.55 \times 2.397 = 9.289 \text{ m}^3/\text{sec}$

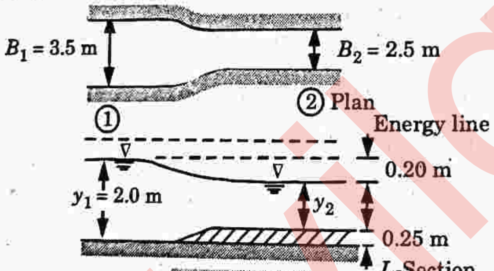


Fig. 2.3.1.

Que 2.4. **Uniform flow occurs at a depth of 1.5 m in a long rectangular channel 3 m wide and laid to a slope of 0.0009. If Manning's $n = 0.015$. Calculate (a) Maximum height of hump on the floor to produce critical depth (b) The width of contraction which will produce critical depth without increasing the upstream depth of flow.**

AKTU 2017-18, Marks 07

Answer

Given : Depth of flow, $y = 1.5$ m, Bed slope, $S = 0.0009$
Width of channel, $b = 3$ m, Manning's constant, $N = 0.015$
To Find : i. Maximum height of hump ii. Width of contraction

1. Critical depth of flow, $y_c = \left(\dfrac{q^2}{g}\right)^{\frac{1}{3}}$

2. Hydraulic mean radius,

$$R = \frac{bd}{b + 2d} = \frac{1.5 \times 3}{3 + 2 \times 1.5} = 0.75$$

3. Velocity of flow, $v = 1/N \times R^{2/3} \times S^{1/2} = 1/0.015 \times 0.75^{2/3} \times 0.0009^{1/2}$

$v = 1.65$ m/sec

4. Discharge, $Q = A \times v = 1.5 \times 3 \times 1.65 = 7.425 \text{ m}^3/\text{sec}$

$q = Q/b = 7.425 / 3 = 2.475$

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{2.475^2}{9.81}\right)^{\frac{1}{3}} = 0.855 \text{ m}$$

5. Maximum height of hump,

$$\Delta z_{max} = y_1 + \frac{v_1^2}{2g} - \frac{3y_c}{2} = 1.5 + \frac{1.65^2}{2 \times 9.81} - \frac{3 \times 0.855}{2}$$

$$\Delta z_{max} = 0.3563 \text{ m}$$

6. $$y_1 + \frac{v_1^2}{2g} = y_c + y_c + \frac{v_c^2}{2g}$$

$$1.5 + \frac{1.65^2}{2 \times 9.81} = y_c + \frac{y_c}{2}$$

$$1.64 = 3\, y_c/2$$

$$1.64 = \frac{3}{2}\left[\frac{q^2}{g}\right]^{\frac{1}{3}}$$

$$12.821 = q^2$$

$$q = 3.58$$

7. We know, $q = Q/b = b = \dfrac{7.425}{3.58}$

8. Contracted width, $b = 2.074$ m

Que 2.5. **A trapezoidal channel has a bottom width of 6 m and side slopes of 1 : 1. The depth of flow is 1.5 m at a discharge of 15 m³/sec. Determine the specific energy. If the critical depth is 0.9 m, discuss the type of flow corresponding to the critical depth.**

AKTU 2016-17, Marks 10

Answer

Given : Bottom width, $B = 6$ m, Side slope = 1 : 1, Depth of flow, $y = 1.5$ m, Discharge, $Q = 15 \text{ m}^3/\text{sec}$, Critical depth, $y_c = 0.9$ m
To Find : Specific energy and type of flow.

1. **Specific Energy :**

$E = y + v^2/2g$

i. Velocity, $v = \dfrac{Q}{A} = \dfrac{Q}{(B + my)y} = \dfrac{15}{(6 + 1 \times 1.5) \times 1.5} = 1.34$ m/sec

ii. $E = 1.5 + (1.34)^2/(2 \times 9.81) = 1.59$ m

2. **Type of Flow :**

i. If critical depth, $y_c = 0.9$ m

ii. Froude number, $F_r = \dfrac{v}{\sqrt{g(A/T)}} = \dfrac{v}{\sqrt{g[(B + my)y/(B + 2my)]}}$

$$F_r = \frac{1.34}{\sqrt{\dfrac{9.81 \times (6 + 1 \times 1.5) \times 1.5}{(6 + 2 \times 1 \times 1.5)}}} = 0.38$$

3. Since the value of Froude number is less than 1. The flow will be subcritical flow.

PART-2

Equation of Gradually Varied Flow and its Limitations.

CONCEPT OUTLINE

Gradually Varied Flow : A steady non-uniform flow in a prismatic channel with gradual changes in its water surface elevation is termed as gradually varied flow (GVF). In a GVF, the velocity varies along the channel and consequently the bed slope, water surface slope, and energy slope will all differ from each other.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.6. **What do you mean by gradually varied flow (GVF) ? State the assumptions and limitations of GVF.**

Answer

A. **Gradually Varied Flow :**

1. A steady non-uniform flow in a prismatic channel with gradual changes in its water surface elevation is termed as gradually varied flow (GVF).
2. In a GVF, the velocity varies along the channel and consequently the bed slope, water surface slope, and energy slope will all differ from each other.

B. **Assumptions :** Following are the assumptions used in analysis of GVF :

1. The bed slope of the channel is small.
2. The energy correction factor, α is unity.
3. The channel is prismatic.
4. Accelerative effect is negligible and hence hydrostatic pressure distribution prevails over channel cross-section.
5. The flow is steady and hence discharge is constant.
6. The roughness coefficient is constant for the length of the channel.

C. **Limitations of Gradually Varied Flow :** Following are the limitations of GVF :

1. Roughness coefficient is not constant for length of channel. It means roughness is not independent.
2. The energy and momentum correction factors are not always unity.
3. The flow may not always steady.
4. The velocity distribution may not be uniform.
5. The pressure distribution may not be hydrostatic even entire channel section.

Que 2.7. **Derive the dynamic equation of GVF, state its various assumptions, also give the limitations of GVF.**

AKTU 2014-15, Marks 10

OR

Using basic differential equation of gradually varied flow (GVF). Show that dy/dx is positive for S_1, M_3 and S_3 profiles.

AKTU 2017-18, Marks 07

OR

Derive the dynamic equation of gradually varied flow.

AKTU 2014-15, Marks 05

Answer

A. **Derivation :**

1. Consider the total energy H of a gradually varied flow in a channel of small slope and α = 1.0 and specific energy E.

$$H = z + E = z + y + (v^2/2g) \quad ...(2.7.1)$$

2. Schematic diagram of a gradually varied flow is shown in Fig. 2.7.1.

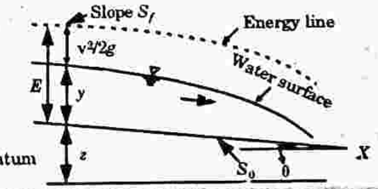


Fig. 2.7.1. Schematic sketch of GVF.

3. Since the water surface, in general varies in the longitudinal (X) direction, the depth of flow and total energy are functions of x.
4. Now, differentiating eq. (2.7.1) wrt x, we get

$$dH/dx = dz/dx + dE/dx \quad ...(2.7.2)$$

or
$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{v^2}{2g}\right) \quad ...(2.7.3)$$

where,

$dH/dx = -S_f$ = Energy slope (decreases in the direction of motion) ...(2.7.4)

$dz/dx = -S_0$ = Bottom slope (decreasing in the downstream direction) ...(2.7.5)

dy/dx = Water surface slope (relative to the bottom of the channel)

iii.
$$\frac{d}{dx}\left(\frac{v^2}{2g}\right) = \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right)\frac{dy}{dx} = -\frac{Q^2}{gA^3}\frac{dA}{dy}\frac{dy}{dx}$$

Since, $dA/dy = T$

$$\therefore \quad \frac{d}{dx}\left(\frac{v^2}{2g}\right) = -\frac{Q^2T}{gA^3}\frac{dy}{dx} \quad ...(2.7.6)$$

4. Eq. (2.7.3) can be written as, $-S_f = -S_0 + \frac{dy}{dx} - \left(\frac{Q^2T}{gA^3}\right)\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1-(Q^2T/gA^3)} \quad ...(2.7.7)$$

5. This forms the basic differential equation of GVF and known as the dynamic equation of GVF. If the value of kinetic energy correction factor is greater than one, then $\frac{dy}{dx} = \frac{S_0 - S_f}{1-(\alpha Q^2T/gA^3)}$ ...(2.7.8)

B. For GVF Profile :

1. The basic differential equation in terms of conveyance (k) and section factor (Z) can be written as,

$$\frac{dy}{dx} = S_o \frac{1-(k_0/k)^2}{1-(Z_c/Z)^2} \quad ...(2.7.9)$$

where,
k = Conveyance at any depth y.
k_0 = Conveyance at normal depth y_0.
Z_c = Section factor at the critical depth y_c.
Z = Section factor at depth y.

2. From eq. (2.7.9), dy/dx will be positive if,

i. $k > k_0$ and $Z > Z_c$ ii. $k < k_0$ and $Z < Z_c$

3. **For S_1 Profile :**
Condition for S_1 profile, $y > y_c > y_0$ i.e., $k > k_0$ and $Z > Z_c$
Hence for S_1 profile, dy/dx will be positive.

4. **For M_3 Profile :**
Condition for M_3 profile, $y_0 > y_c > y$ i.e., $k_0 > k$ and $Z < Z_c$
Hence for M_3 profile dy/dx will be positive

5. **For S_3 Profile :**
Condition for S_3 profile, $y_c > y_0 > y$ i.e., $k_0 > k$ and $Z < Z_c$
Hence for S_3 profile dy/dx will be positive.

C. Assumptions and Limitations : Refer Q. 2.6, Page 2–9B, Unit-2.

Que 2.8. **Show that the differential equation of gradually varied flow in a rectangular channel of variable width B can be expressed as**

$$\frac{dy}{dx} = \frac{S_o - S_f + \left(\frac{Q^2y}{gA^3}\frac{dB}{dx}\right)}{1-\frac{Q^2B}{gA^3}}$$

Answer

1. The GVF equation is $\frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{v^2}{2g}\right)$
where, $dH/dx = -S_f$, $dz/dx = -S_0$

2. $\therefore \quad -S_f = -S_0 + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{Q^2}{2gA^2}\right)$ $(\because Q = AV)$

$$S_0 - S_f = \frac{dy}{dx} + \frac{d}{dx}\left(\frac{Q^2}{2g \times B^2y^2}\right) \quad (\because A = Ay)$$

$$S_0 - S_f = \frac{dy}{dx} + \frac{d}{dy}\left(\frac{Q^2}{2gy^2B^2}\right) \times \frac{dy}{dx} + \frac{d}{dB}\left(\frac{Q^2}{2gB^2y^2}\right)\frac{dB}{dx}$$

$$S_0 - S_f = \frac{dy}{dx} - \frac{Q^2B}{gA^3}\frac{dy}{dx} - \frac{Q^2y}{gA^3}\frac{dB}{dx} \Rightarrow \frac{dy}{dx} = \frac{S_0 - S_f + \frac{Q^2y}{gA^3}\frac{dB}{dx}}{1-\frac{Q^2B}{gA^3}}$$

Que 2.9. **Show that for horizontal, frictionless rectangular channel of varying width B_1, $(1-F_r^2)\frac{dy}{dx} - F_r^2\left(\frac{y}{B}\right)\frac{dB}{dx} = 0$**
Where, F_r = Froude number.

Answer

1. We known that, $\frac{dy}{dx} = \frac{S_0 - S_f + \frac{Q^2y}{gA^3}\frac{dB}{dx}}{1-\frac{Q^2B}{gA^3}}$ ...(2.9.1)

2. For horizontal, frictionless rectangular channel, $S_0 = 0$ and $S_f = 0$

Hence,
$$\frac{dy}{dx} = \frac{\frac{Q^2y}{gA^3}\frac{dB}{dx}}{1-\frac{Q^2B}{gA^3}}$$

3. We know that, Froude number, $F_r = \dfrac{v}{\sqrt{g(A/T)}}$

$F_r^2 = v^2/gy$ [$\because$ For rectangular channel, $(A/T) = y$]

$$F_r^2 = \frac{Q^2}{gy^3B^2} = \frac{Q^2B}{g(y^3B^3)} = \frac{Q^2B}{gA^3}$$

4. Value of S_0, S_f and (Q^2B/gA^3) put in eq. (2.9.1), we get

$$\therefore \quad \frac{dy}{dx} = \frac{F_r^2\left(\dfrac{y}{B}\right)\dfrac{dB}{dx}}{1-F_r^2} \Rightarrow (1-F_r^2)\frac{dy}{dx} - F_r^2\left(\frac{y}{B}\right)\frac{dB}{dx} = 0$$

Que 2.10. **Prove that for a wide rectangular channel, if the Manning's formula is used, the differential equation of GVF becomes**

$$\frac{dy}{dx} = S_0\left[\frac{1-(y_0/y)^{\frac{10}{3}}}{1-(y_c/y)^3}\right]$$

Answer

1. Basic GVF equation is, $\dfrac{dy}{dx} = \dfrac{S_0 - S_f}{1-\dfrac{Q^2T}{gA^3}} = \dfrac{S_0\,[1-S_f/S_0]}{1-\dfrac{Q^2T}{gA^3}}$

2. And also we know that, $\dfrac{dy}{dx} = S_0\left[\dfrac{1-(k_0/k)^2}{1-(Z_c/Z)^2}\right]$ ...(2.10.1)

3. Where, $k = (1/n)\,AR^{2/3} = (1/n)\,By^{5/3}$

4. Similarly for k_0, $k_0 = (1/n)\,By_0^{5/3}$, hence $k/k_0 = (y_0/y)^{5/3}$ ...(2.10.2)

5. We know that, $Z^2 = C_1\,y^M$ ...(2.10.3)

where, M = First hydraulic exponent.

6. For rectangular channel, $M = 3$, value of M put in eq. (2.10.3), we get

$\therefore$ $Z^2 = C_1y^3$ and $Z_c^2 = C_1y_c^3$

or $(Z_c/Z)^2 = (y_c/y)^3$ ...(2.10.4)

7. Putting these values in eq. (2.10.1), we get

$$\frac{dy}{dx} = S_0\left[\frac{1-(y_0/y)^{10/3}}{1-(y_c/y)^3}\right]$$

Que 2.11. **Show that, for a wide rectangular channel, if Chezy's formula is used, the differential equation of GVF is given by,**

$$\frac{dy}{dx} = S_0\left[\frac{1-(y_0/y)^3}{1-(y_c/y)^3}\right]$$

Answer

1. Basic GVF equation is, $\dfrac{dy}{dx} = S_0\left[\dfrac{1-S_f/S_0}{1-Q^2T/gA^3}\right]$ ...(2.11.1)

2. By Chezy's equation, $Q = CA\sqrt{RS_f} = CBy\sqrt{yS_f}$ ...(2.11.2)

3. For rectangular channel $Q_0 = Q$ and For wide rectangular channel $R = y$

Normal discharge, $Q_0 = CBy_0\sqrt{y_0S_0}$ ...(2.11.3)

4. Dividing eq. (2.11.2) by (2.11.3), we get

$S_f/S_0 = (y_0/y)^3$ ...(2.11.4)

$Q^2T/gA^3 = (y_c/y)^3$ [$\because$ $(Q^2/gB^2) = y_c^3$] ...(2.11.5)

5. Putting these values in eq. (2.11.1), we get

$$\frac{dy}{dx} = S_0\left[\frac{1-(y_0/y)^3}{1-(y_c/y)^3}\right]$$

Que 2.12. **Integrate the differential equation of GVF for a horizontal channel to get the profile equation as**

$$x = \frac{h_c}{S_c}\left[\frac{\left(\dfrac{h}{h_c}\right)^{N-M+1}}{N-M+1} - \frac{\left(\dfrac{h}{h_c}\right)^{N+1}}{N+1}\right] + \text{Constant.}$$

AKTU 2017-18, Marks 07

Answer

1. We know, $\dfrac{dh}{dx} = \dfrac{S_0 - S_f}{1-\dfrac{Q^2T}{gA^3}}$ ...(2.12.1)

2. For a horizontal channel, $S_0 = 0$

Also, $S_f = \dfrac{Q^2}{K^2} = \dfrac{K_c^2\,S_c}{K^2} = \left(\dfrac{h_c}{h}\right)^N S_c$ ...(2.12.2)

Also; $\dfrac{Q^2}{g} = h_c^2$ and $\dfrac{A^3}{T} = h^2$

Thus, $\dfrac{Q^2T}{gA^3} = \left(\dfrac{h_c}{h}\right)^M$ ...(2.12.3)

3. With the half of eq. (2.12.2) and eq. (2.12.3), eq. (2.12.1) can be written as,

$$\frac{dh}{dx} = \frac{-(h_c/h)^N\,S_c}{1-(h_c/h)^M}$$

$$-dx\,S_c = \left[\frac{1-(h_c/h)^M\,dh}{(h_c/h)^N}\right]$$

$$-dx\,S_c = \left[\left(\frac{h}{h_c}\right)^N - \left(\frac{h}{h_c}\right)^{N-M}\right]dh \quad ...(2.12.4)$$

4. Integrating eq. (2.12.4) on both side, we get

$$-S_c x = h_c\left[\frac{(h/h_c)^{N+1}}{N+1} - \frac{(h/h_c)^{N-M+1}}{N-M+1}\right] + \text{Constant}$$

$$x = \frac{h_c}{S_c}\left[\frac{(h/h_c)^{N-M+1}}{N-M+1} - \frac{(h/h_c)^{N+1}}{N+1}\right] + \text{Constant}$$

PART-3

Flow Classification and Surface Profiles, Integration of Varied Flow Equation by Analytical, Graphical and Numerical Methods.

CONCEPT OUTLINE

Flow Classification : The flow in channels can be classified into following types depending upon the change in depth of flow with respect to space and time.

1. Steady and unsteady flow.
2. Uniform and non-uniform flow.
3. Laminar and turbulent flow.
4. Subcritical, critical and supercritical flow.

Types of Surface Profiles : The various water surface profiles occurring in the channels are designated with reference to the bottom slopes of the channels. The surface profiles which occur in mild-sloped channels are known as *M*-curves; those which occur in steep-sloped channels are known as *S*-curves; those which in critical-sloped channels are known as *C*-curves; those which occur in horizontal channels are known as *H*-curves, and those which occur in adverse-sloped channels are known as *A*-curves.

Method of computations for GVF :

1. Analytical method as Integration method.
2. Numerical method.
3. Graphical method.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.13. **Classify the following open-channel flow situations :**

1. **Flow from a sluice gate.**
2. **Flow in a main irrigation canal.**
3. **A river during flood.**

AKTU 2015-16, Marks 10

Answer

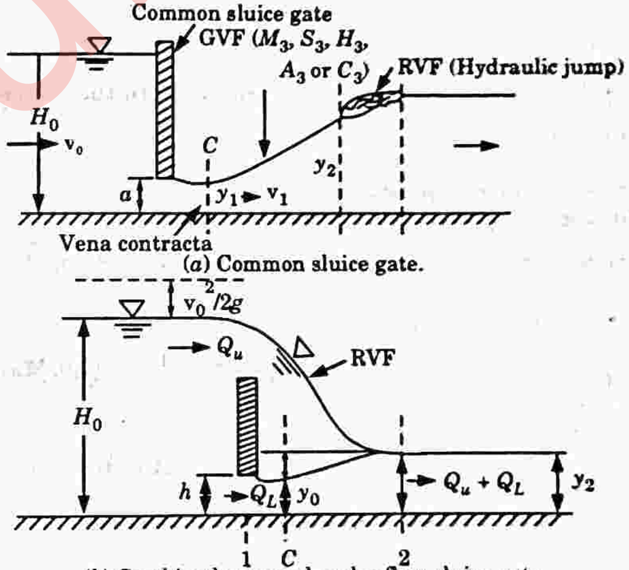
1. **Flow from Sluice Gate :**

i. In a common sluice gate, water flows below the sluice gate.

ii. Depending upon the slope of bed of the channel or stream on which sluice gate is operating flow profile just downstream of the opening, from vena contracta may be M_3, S_3, H_3, A_3 and C_3 types gradually varied flow (GVF).

iii. The small portion of the profile beyond the GVF profiles is the Rapid Varied Flow (RVF) profile, *i.e.*, the hydraulic jump portion.

iv. But in practice, in some cases, sluice gates with combined over and under flow are used as control gates in canal which are called under flow gates as shown in Fig. 2.13.1(*b*). In such sluice gate, flow profile over them is Rapidly Varied Flow (RVF).



(*a*) Common sluice gate.

(*b*) Combined over and under flow sluice gate.

Fig. 2.13.1. Sluice gates in two different situations.

2. **Flow in a Main Irrigation Canal :**

i. Open channel flow can be classified in many ways. A very common classification of open channel (gravity) flow is according to the change of flow depth with respect to time and space.

ii. Steady flow is the flow in a channel where the water depth either does not change with time or can be assumed to be constant during the time interval under consideration.

iii. Unsteady flow is surge waves in irrigation canals after a sudden opening or closing of a gate.

- Open channel flow
  - Steady
    - Uniform flow
    - Gradually varied flow (GVF)
    - Rapidly varied flow (RVF)
    - Spatially varied flow (SVF)
  - Unsteady
    - Gradually varied uniform flow (GVUF)
    - Rapidly varied uniform flow (RVUF)
    - Spatially varied uniform flow (SVUF)

3. **A River During Flood :**

i. The flood flow in a river is a typical GVUF.

ii. The continuity equation in its general form for GVUF is,

$$A\frac{\partial v}{\partial x} + vT\frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} - q + v\left(\frac{\partial A}{\partial x}\right)_y = 0 \quad \text{...(2.13.1)}$$

iii. The equation of motion of GVUF is,

$$\frac{\partial y}{\partial x} + \frac{v}{g}\frac{\partial v}{\partial x} + \frac{1}{g}\frac{\partial v}{\partial t} = S_0 - S_f \quad \text{...(2.13.2)}$$

iv. The continuity eq. (2.13.1) and the equation of motion of unsteady open-channel flow eq. (2.13.2) are commonly known as Saint-Venant equations.

Que 2.14. **Classify the various flow profiles with the help of their neat sketches.**

OR

Sketch the GVF profile produced on

i. Mild slope. **ii. Steep slope.**

iii. Critical slope. **AKTU 2014-15, Marks 10**

OR

Sketch the GVF profiles produced on :

i. Steep Slope, and ii. Critical slope. **AKTU 2018-19, Marks 07**

Answer

1. For a given channel, normal depth y_0 and critical depth y_c are two fixed depths if Q, S_0 and n are fixed.
2. Based on this, the channels are classified into five categories :

Table 2.14.1. Classification of flow profile.

S. No.	Channel Category	Symbol	Characteristic Condition	Remark
1.	Mild slope	M	$y_0 > y_c$	Subcritical flow at normal depth.
2.	Steep slope	S	$y_c > y_0$	Supercritical flow at normal depth.
3.	Critical slope	C	$y_c = y_0$	Critical flow at normal depth.
4.	Horizontal bed	H	$S_0 = 0$	Cannot sustain uniform flow.
5.	Adverse slope	A	$S_0 < 0$	Cannot sustain uniform flow.

3. Also there are two cases where y_0 does not exist, *i.e.*, when

i. The channel bed is horizontal, $(S_0 = 0)$,

ii. The channel has an adverse slope, (S_0 is negative).

4. For each of the five categories of channels, lines representing the critical depth and normal depth (if exists) can be drawn in the longitudinal section.
5. These would divide the whole flow space into three regions as :
   Region 1 : Space above the top most line.
   Region 2 : Space between the top line and the next lower line.
   Region 3 : Space between the second line and the bed.

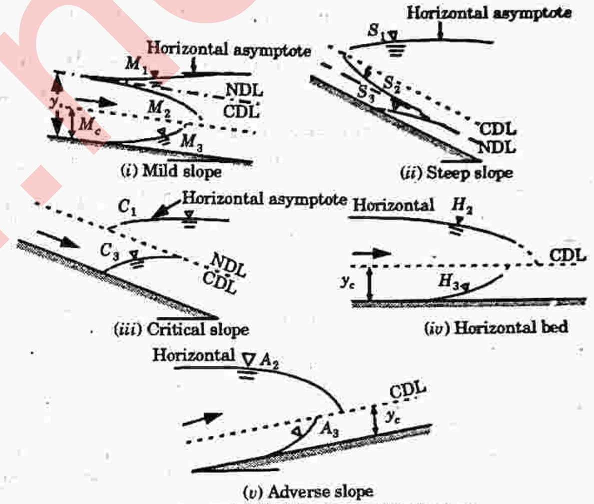


Fig. 2.14.1. Various GVF profiles.

6. The various possible gradually varied flow profiles are tabulated below :

Table 2.14.2. Types of GVF profiles.

Channel	Region	Condition	Type
Mild slope	1	$y > y_0 > y_c$	M_1
	2	$y_0 > y > y_c$	M_2
	3	$y_0 > y_c > y$	M_3
Steep slope	1	$y > y_c > y_0$	S_1
	2	$y_c > y > y_0$	S_2
	3	$y_c > y_0 > y$	S_3
Critical slope	1	$y > y_0 = y_c$	C_1
	3	$y < y_0 = y_c$	C_3
Horizontal bed	2	$y > y_c$	H_2
	3	$y < y_c$	H_3
Adverse slope	2	$y > y_c$	A_2
	3	$y < y_c$	A_3

7. The characteristic shapes and end conditions of all these profiles are shown in Fig. 2.14.1.

i. It is evident from figure that all the curves in region 1 have positive slopes these are commonly known as backwater curves.

ii. Similarly all the curves in region 2 have negative slopes and are referred to as drawdown curves.

iii. At critical depth the curves are indicated by dashed lines to remind that the GVF equation is strictly not applicable in that neighbourhood.

Que 2.15. **A rectangular channel with a bottom width of 4.0 m and a bottom slope of 0.0008 has a discharge of 1.50 m³/sec. In gradually varies flow in this channel, the depth at certain location is found to be 0.30 m. Assuming $n = 0.016$, determine the type of GVF profile.**

AKTU 2014-15, Marks 10

Answer

Given : Depth, $y = 0.3$ m, Bottom width, $B = 4$ m,
Bottom slope, $S_o = 0.0008$, Discharge, $Q = 1.5$ m³/sec
Manning coefficient, $n = 0.016$
To Find : Type of GVF profile.

1. Discharge per metre width, $q = Q / B = 1.5 / 4 = 0.375$ m³/sec/m
2. Critical depth is given by, $y_c = (q^2/g)^{1/3} = [(0.375)^2 / 9.81]^{1/3} = 0.243$ m
3. Normal depth can be calculated by, $q = (1/n)\, y_0^{5/3}\, S_0^{1/2}$

$$0.375 = (1/0.016) \times (y_o)^{5/3} \times (0.0008)^{1/2}$$

Normal depth, $y_0 = 0.397$ m

4. Since, $y_0 > y > y_c$ condition follows. Hence, there is M_2 type of GVF profile.

Que 2.16. **Discuss the features of flow profiles.**

Answer

1. **Type-*M* Profiles :** Following are three types of *M* profile :

i. **M_1 Profile :**

a. The most common of all GVF profiles is the M_1 type, which is a subcritical flow condition.

b. Obstructions to flow, such as weirs, dams, control structures and natural features, such as bends, produce M_1 backward curves.

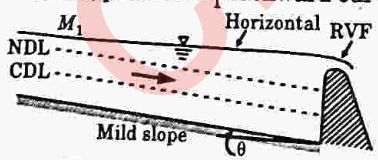


Fig. 2.16.1. M_1 profile.

ii. **M_2 Profiles :** These profiles occur at a sudden drop in the bed of the channel, at construction type of transitions and at the canal outlet into pools.

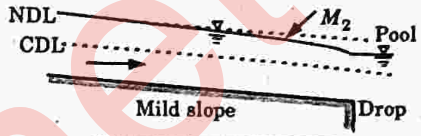


Fig. 2.16.2. M_2 profile.

iii. **M_3 Profiles :**

a. M_3 types profile occurs, where a supercritical stream enters a mild-slope channel.

b. The flow leading from a spillway or a sluice gate to a mild slope forms a typical example.

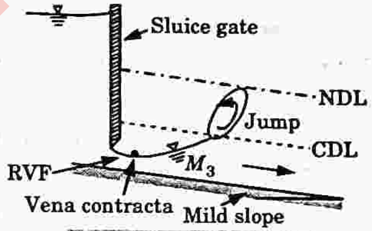


Fig. 2.16.3. M_3 profile.

c. The beginning of the M_3 curve is usually followed by a small stretch of rapidly-varied flow and downstream is generally terminated by a hydraulic jump.

d. Compared to M_1 and M_2 profiles, M_3 curves are of relatively short length.

2. **Type-*S* Profiles :** Following are the three types of *S* profiles :

i. **S_1 Profile :**

a. The S_1 profile is produced when the flow from a steep channel is terminated by a deep pool created by an obstruction, such as a weir or dam.

b. At the beginning of the curve, the flow changes from the normal depth (supercritical flow) to subcritical flow through a hydraulic jump.

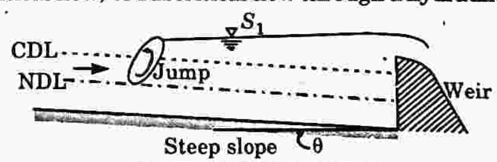


Fig. 2.16.4. S_1 profile.

ii. **S_2 Profile :**

a. Profile of the S_2 type occur at the entrance region of a steep channel leading from a reservoir and at a break of grade from mild slopes to steep slope as shown in Fig. 2.16.5.

b. Generally S_2 profiles are of short length.

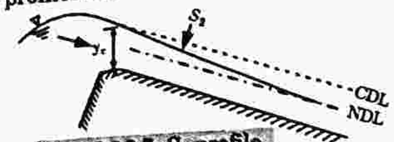


Fig. 2.16.5. S_2 profile.

iii. S_3 Profile :

a. The S_3 curve also results when a flow exists from a steeper slope to a less steep slope as shown in Fig. 2.16.6.

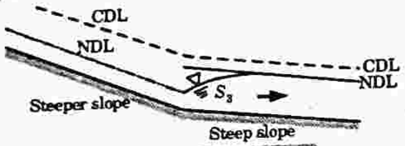


Fig. 2.16.6. S_3 profile.

3. **Type C Profiles :** C_1 and C_3 profiles are very rare and are highly unstable.

4. **Type H Profiles :**

a. A horizontal channel can be considered as the lower limit reached by a mild slope as its bed slope becomes flatter.

b. It is obvious that there is no region 1 for a horizontal channel as $y_0 = \infty$.

c. The H_2 and H_3 profiles are similar to M_2 and M_3 profiles respectively as shown in Fig. 2.16.7. However, the H_2 curve has a horizontal asymptote.

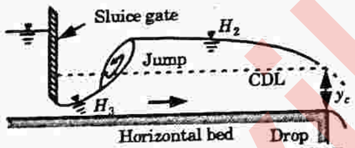


Fig. 2.16.7. H_2 and H_3 profiles.

5. **Type A Profile :**

a. Adverse slopes are rather rare and A_2 and A_3 curves are similar to H_2 and H_3 curves respectively as shown in Fig. 2.16.8.

b. These profiles are of very short length.

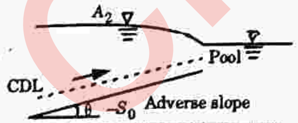


Fig. 2.16.8. A_2 profile.

Que 2.17. **How you will define transitions between sub critical flow and super critical flow ? Also draw the diagram.**

AKTU 2017-18, Marks 07

Answer

Transition from Sub-critical to Super-critical Flow :

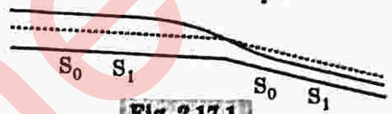


Fig. 2.17.1.

1. When the flow changes from subcritical to supercritical the water surface lowers gradually from a higher depth to a lower depth by passing through critical depth.

2. In the region where the flow changes from subcritical to critical flow, a gradually varied flow takes place.

Que 2.18. **A rectangular channel 10 m wide is laid with a break in its bottom slope from 0.01 to 0.0064. If it carries 125 m³/sec, determine the nature of the surface profile and compute its length. Take n = 0.015.**

AKTU 2016-17, Marks 10

Answer

Given : Width of Bed, B = 10 m, Slope, $S_{01} = 0.01$, $S_{02} = 0.0064$
Discharge, Q = 125 m³/sec, Manning's coefficient, $n = 0.015$
To Find : Surface profile and length of profile.

A. Nature of surface profile :

1. Discharge per unit width, $q = Q / B = 125 / 10 = 12.5$ m³/sec/m

Critical depth, $$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{12.5}{9.81}\right)^{\frac{1}{3}} = 1.084 \text{ m}$$

2. For normal depth calculation :

$$\phi = \frac{Q \times n}{\sqrt{S_0} \times B^{8/3}} = \frac{125 \times 0.015}{\sqrt{S_0} \times (10)^{8/3}} = \frac{4.04 \times 10^{-3}}{\sqrt{S_0}}$$

3. Using this relation, the normal depth for various cases are calculated as below. [Value of $\frac{y_0}{B}$ are fixed, mug up these values]

S_0	ϕ	y_0/B	y_0
0.01	0.0404	0.16	1.6
0.0064	0.0505	0.19	1.9

$$\frac{y_0}{B} = 0.16 + \frac{(0.165 - 0.16)}{(0.04104 - 0.03919)} \times (0.0404 - 0.03919) = 0.16$$

y_{01}(m)	y_{02}(m)	y_c(m)	Type of Grade Change	Possible Types of Profiles
1.6	1.9	1.084	Mild to milder mild	M_1

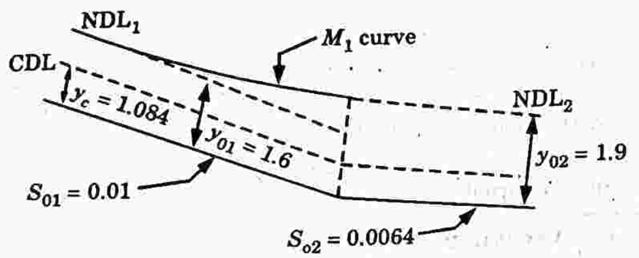


Fig. 2.18.1.

B. Length of Profile :

1. Velocity of flow at section 1, $v_1 = \dfrac{125}{10 \times 1.6} = 7.8125$ m/sec

2. Specific energy, $E_1 = y_1 + \dfrac{v_1^2}{2g} = 1.6 + \dfrac{(7.8125)^2}{2 \times 9.81} = 4.71$ m

3. Hydraulic radius, $R_1 = \dfrac{A}{P} = \dfrac{By_0}{2y_0 + B} = \dfrac{10 \times 1.6}{2 \times 1.6 + 10} = 1.212$ m

4. Energy slope, $S_{f1} = \dfrac{v_1^2 n^2}{R_1^{4/3}} = \dfrac{(7.8125 \times 0.015)^2}{(1.212)^{4/3}} = 0.0106$

5. Depth of flow at section 2, $y_2 = 1.9$ m

6. Velocity, $v_2 = \dfrac{125}{10 \times 1.9} = 6.58$ m/sec

7. Specific energy, $E_2 = y_2 + \dfrac{v_2^2}{2g} = 1.9 + \dfrac{6.58^2}{2 \times 9.81} = 4.107$ m

8. Hydraulic radius, $R_2 = \dfrac{10 \times 1.9}{2 \times 1.9 + 10} = 1.377$ m

9. Energy slope, $S_{f2} = \dfrac{v_2^2 n^2}{R_2^{4/3}} = \dfrac{(6.58 \times 0.015)^2}{(1.377)^{4/3}}$, $S_{f2} = 0.0064$

10. Average energy slope, $S_f = \dfrac{S_{f1} + S_{f2}}{2} = \dfrac{0.0106 + 0.0064}{2} = 0.0085$

11. Length of profile, $L = \dfrac{4.71 - 4.107}{S_0 - S_f} = \dfrac{4.71 - 4.107}{0.01 - 0.0085} = 402$ m

Hence, length of M_1 profile is 402 m.

Que 2.19. **Explain the various methods of computations for GVF profile.**

Answer

The various available methods for computing GVF profile can be classified as :

A. Analytical Method :

1. Differential equation of GVF for prismoidal channel is given by,

$$\frac{dy}{dx} = S_0 \times \frac{1 - (k_0^2 / k^2)}{1 - (Z_C^2 / Z^2)} = F(y) \quad ...(2.19.1)$$

which is non-linear, first order differential equation.

2. Let it is required to find $y = f(x)$ in the depth range y_1 to y_2 the following assumptions are made.

i. Conveyance at any depth y is given by,

$$k^2 = C_2 y^N \quad ...(2.19.2)$$

At depth y_0, $$k_0^2 = C_2 y_0^N \quad ...(2.19.3)$$

This implies C_2 and N are constants.

ii. The section factor Z is given by,

At any depth y, $$Z^2 = C_1 y^M \quad ...(2.19.4)$$

At critical depth y_c, $$Z_C^2 = C_1 y_C^M \quad ...(2.19.5)$$

This implies coefficient C_1 and the first hydraulic exponent M are constant.

3. Substituting the value of eq. (2.19.2), (2.19.3), (2.19.4) and (2.19.5) in eq. (2.19.1), we get

$$\frac{dy}{dx} = S_0 \frac{1 - (y_0 / y)^N}{1 - (y_c / y)^M} \quad ...(2.19.6)$$

4. Let us consider $u = (y / y_0) \Rightarrow dy = y_0\, du$

5. Put, $u = y / y_0$ and $dy = y_0 du$ in eq. (2.19.6), we get

$$\frac{du}{dx} = \frac{S_0}{y_0}\left[\frac{1 - \dfrac{1}{u^N}}{1 - \left(\dfrac{y_c^M}{y_0^M}\right)\dfrac{1}{u^M}}\right]$$

Therefore, $$dx = \frac{y_0}{S_0}\left[1 - \frac{1}{1 - u^N} + \left(\frac{y_c}{y_0}\right)^M \frac{u^{N-M}}{1 - u^N}\right]du$$

6. On integration, we get

$$x = \frac{y_0}{S_0}\left[u - \int_0^u \frac{du}{1 - u^N} + \left(\frac{y_c}{y_0}\right)^M \int_0^u \frac{u^{N-M}}{1 - u^N}\, du\right] + \text{constant} \quad ...(2.19.7)$$

Take, $$\int_0^u \frac{du}{1 - u^N} = F(u, N)$$

7. Also take, $V = u^{\frac{N}{J}}$ where, $J = \dfrac{N}{N - M + 1}$

$$dV = \frac{N}{J} u^{\frac{N}{J}-1} du = (N - M + 1) u^{N-M} du$$

$$\int_0^u \frac{u^{N-M}}{1-u^N} du = \frac{1}{N-M+1} \int_0^v \frac{dV}{1-V^J} = \frac{J}{N} F(V, J)$$

8. Here we notice that $F(V, J)$ is the same function as $f(u, N)$ with, u and N replaced V and J respectively.
9. So eq. (2.19.7) can be written as

$$x = \frac{y_0}{S_0}\left[u - F(u, N) + \left(\frac{y_c}{y_0}\right)^M \frac{J}{N} F(V, J)\right] \quad ...(2.19.8)$$

The function $f(u, N)$ is called as varied flow function.

10. For a wide rectangular channel if the Chezy's formula with C = constant is used, the hydraulic exponent will be $M = 3.0$ and $N = 3.0$. Putting these values in eq. (2.19.8), we get

$$x = \frac{y_0}{S_0}\left[u - 1 - \left(\frac{y_c}{y_0}\right)^3 F(u, 3)\right] + \text{constant} \quad ...(2.19.9)$$

Since $\quad J = \dfrac{N}{N-M+1} = \dfrac{3}{3-3+1} = 3$

where $\quad F(u, 3) = \displaystyle\int_0^u \frac{du}{1-u^3}$

$F(u, 3)$ is called Bresse's function.

B. Numerical Method : The numerical method is broadly classified into two categories :

1. **Simple Numerical Methods :** They were developed for simple calculation where computation can be done by hand. Two commonly used simple numerical methods are :

i. **Direct Step Method :** This method is suitable for prismatic channels.

We know that $\quad dE / dx = S_0 - S_f$

It can also be written in finite difference form

$$\Delta E / \Delta x = S_0 - \bar{S}_f \quad ...(2.19.10)$$

where, $\bar{S}_f$ is the average friction slope, $\Delta x = \Delta E / (S_0 - \bar{S}_f)$

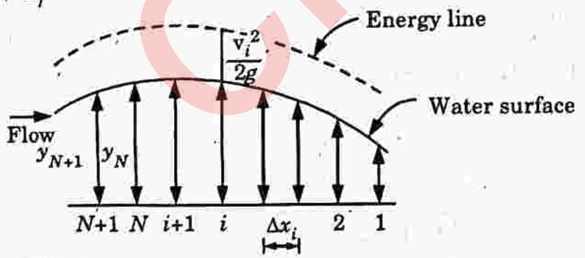


Fig. 2.19.1. Sketch for direct step method.

ii. Taking two sections 1 and 2,

$$x_2 - x_1 = \frac{E_2 - E_1}{S_0 - \frac{1}{2}(S_{f1} - S_{f2})} \quad ...(2.19.12)$$

iii. Consider the above water surface profile and it is divided into N parts and each part is of different depth.

$$\Delta E = \Delta\left(y + \frac{v^2}{2g}\right) = \Delta\left(y + \frac{Q^2}{2gA^2}\right)$$

$$\Delta E = E_{i+1} - E_i = \left(y_{i+1} + \frac{Q^2}{2gA_{i+1}^2}\right) - \left(y_i + \frac{Q^2}{2gA_i^2}\right)$$

$$\bar{S}_f = \frac{1}{2}(S_{fi+1} - S_{fi}) = \frac{n^2Q^2}{2}\left[A_{i+1}^{\frac{1}{2}} R_{i+1}^{\frac{4}{3}} + A_i^{\frac{1}{2}} R_i^{\frac{4}{3}}\right]$$

For general, $\quad \Delta x_i = \dfrac{E_{i+1} - E_i}{S_0 - \bar{S}_f} \quad ...(2.19.13)$

2. **Advanced Numerical Method :** This method is normally suited for use in computer.

i. Basic equation of GVF, $\dfrac{dy}{dx} = \dfrac{S_0 - S_f}{1 - (Q^2T / gA^3)}$

keeping S_0, n, Q and channel geometry constant the differential equation is a function of y and can be written as

$$\frac{dy}{dx} = f(y) \text{ or } f(y) = \frac{S_0 - S_f}{1 - (Q^2T / gA^3)}$$

Using notation $\quad y_i = y(x_i)$ means y at x_i

$y_{i+1} = y(x_{i+1})$ means y at x_{i+1}

$x_{i+1} = x_i + \Delta x$

ii The various numerical methods are as follows :

a. **Standard Fourth Order Runge-Kutta Method (SRK) :**

$$y_{i+1} = y_i + 1/6\,(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = \Delta x\, f(y_i),\ k_2 = \Delta x\ f\left(y_i + \frac{k_1}{2}\right),\ k_3 = \Delta x\ f\left(y_i + \frac{k_2}{2}\right)$$

$$k_4 = \Delta x\ (y_i + k_3)$$

b. **Kutta Merson Method (KM) :**

$$y_{i+1} = y_i + 1/2(k_1 + 4k_4 + k_5)$$

$$k_1 = \frac{1}{3}\Delta x\, f(y_i),\ k_2 = \frac{1}{3}\Delta x\, f(y_i + k_1),\ k_3 = \frac{1}{3}\Delta x\, f\left(y_i + \frac{k_1}{2} + \frac{k_2}{2}\right)$$

$$k_4 = \frac{1}{3}\Delta x\, f\left(y_i + \frac{3}{8}k_1 + \frac{9}{8}k_3\right),\ k_5 = \frac{1}{3}\Delta x\, f\left(y_i + \frac{3}{2}k_1 - \frac{9}{2}k_3 + 6k_4\right)$$

3. **Graphical Method :**

i. We know that, $\quad \dfrac{dy}{dx} = \dfrac{S_0 - S_f}{1 - (Q^2T / gA^3)}$

ii. It can be written as $\dfrac{dy}{dx} = \dfrac{S_0 - \dfrac{\eta^2 Q^2}{A^2 R^{4/3}}}{1 - \dfrac{Q^2 T}{gA^3}}$ or $x_{1,2} = \displaystyle\int_{y_1}^{y_2} \dfrac{1 - \dfrac{Q^2 T}{gA^3}}{S_0 - \dfrac{\eta^2 Q^2}{A^2 R^{4/3}}} dy$

iii. Where, $x_{1,2}$ represents the distance between two sections where the flow depths are y_1 and y_2. The integral can be obtained graphically by plotting the variation of the function within the integral *i.e.*, plotting dy/dx verses y and by calculating area enclosed gives the required result.

Que 2.20. **A natural channel with 50 m width and 1.50 m deep has an average bed slope of 0.0005. Estimate the length of the GVF profile produced by a low weir which raises the water surface just upstream of it by 0.75 m. Assume n = 0.035.**

Answer

Given : Width of channel, B = 50 m, Bed slopes, S_0 = 0.00005
Depth of channel, y = 1.5 m, Manning's coefficient, n = 0.035
To Find : Length of GVF profile.

1. Considering the wide rectangular channel, the discharge per unit width,

$$q = \frac{1}{n} y_0^{\frac{5}{3}} S_0^{\frac{1}{2}} = \frac{1}{0.035} \times (1.50)^{\frac{5}{3}} \times (0.0005)^{1/2} = 1.25 \text{ m}^2/\text{sec}$$

2. Critical depth, $y_c = \left(\dfrac{q^2}{g}\right)^{\frac{1}{3}} = \left(\dfrac{1.25}{9.81}\right)^{\frac{1}{3}} = 0.50$ m

3. Since, $y > y_0 > y_c$ the GVF profile is an M_1 curve with depth of flow y = 1.5 + 0.75 = 2.25 m at the low depth as the control.
4. Calculation of length of GVF profile is shown in Table 2.20.1.

$$R = \frac{50y}{50 + 2y}, \quad S_f = \frac{n^2 v^2}{R^{4/3}}, \quad E = y + \frac{v^2}{2g}$$

$$\overline{S_f} = \frac{S_n + S_{n+1}}{2}, \quad \Delta x = \frac{\Delta E}{S_o - \overline{S}_f}$$

Table 2.20.1.

S.No.	$y(m)$	v(m/sec)	R	E	ΔE	S_f	$\overline{S}_f$	$S_o - \overline{S}_f$	Δx	x
1.	2.25	0.555	2.064	2.266		0.000144				0
					–0.246		0.0001772	0.000323	–761.6	
2.	2.0	0.625	1.852	2.02		0.0002104				761.6
					–0.244		0.0002672	0.000233	–1047.21	
3.	1.75	0.714	1.636	1.776		0.000324				1808.81
					–0.241		0.0004302	0.0000698	–3452.72	
4.	1.50	0.834	1.415	1.535		0.0005364				5261.53

Que 2.21. **A rectangular channel is 20 m wide and carries discharge of 65 m³/sec. It is laid at a slope of 0.0001. At a certain section along the channel length, the depth of flow is 2.0 m. How far will be the depth be 2.60 m ? Take n = 0.02.**

AKTU 2018-19, Marks 07

Answer

Given : Width, b = 20 m, Discharge, Q = 65 m³/sec, S_b = 0.0001,
Depth, y_1 = 2 m, y_2 = 2.6 m, Manning's coefficient, n = 0.02
To Find : Length of channel between depth = 2 m to 2.6 m.

1. Velocity of flow, v = Q / by = 65 / (20 × y) = 3.25/y
2. Hydraulic radius, $R = \dfrac{A}{P} = \dfrac{\text{Area}}{\text{Wetted perimeter}} = \dfrac{b \times y}{b + 2y} = \dfrac{20y}{20 + 2y}$
3. Let the profile be computed in 2 steps :

$y(m)$	v = 3.25/y (m/sec)	$R = \dfrac{20y}{20+2y}$	$E = y + \dfrac{v^2}{2g}$	$S_f = \dfrac{(vn)^2}{R^{4/3}}$	$\overline{S}_f$	$\Delta x = \dfrac{E_2 - E_1}{S_b - \overline{S}_f}$
2	1.625	1.667	2.135	0.00054		
2.3	1.413	1.87	2.402	0.00035	0.00045	762.86
2.6	1.25	2.0645	2.68	0.00024	0.000295	1425.65

4. Length, $L = (\Delta x_1 + \Delta x_2)$ = (762.86 + 1425.65) = 2188.51 m
Length, L = 2.189 km

PART-4

Flume, Venturi Flume, Parshall Flume, Broad-Crested Weir, Standing Wave Flume, Current Meter and Floats.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.22. **Define flume and classify them. Also give the advantages of flume.**

Answer

Flumes :

1. Flumes are flow measuring devices that works on the principle of forming a critical depth in the channel by either utilizing a drop or by constricting the channel.
2. Flume is a channel usually supported on or above the surface of the ground to carry water.
3. Flumes are specially shaped, engineered structures that are used to measure the flow of water in open channels.

Classification of Flume :

1. Flume with a vertical drop :
i. Standing wave flume. ii. Venturi flume.
2. Flume with a constricted section :
i. **Short-throated Flume :** Short does not refer to the flume length but to the fact that flow is controlled in a very specific region of the flume to produce the level-to-flow relationship.
 a. Parshall flume. b. Montana flume (modified Parshall).
 c. USGS Portable flume (modified Parshall).
 d. HS / H / HL-flume. e. Trapezoidal flume.
 f. Cut-throat flume.
ii. **Long-throated Flume :** Long-throated flumes control flow in a throat that is long enough to cause parallel flow lines in the section of flow control :
 a. Palmer-Bowlus flume. b. RBC flume.

Advantages : Following are the various advantages of flume :

1. Flume measure higher flow rates than a comparably sized weir.
2. Less head loss.
3. The ability to pass debris more readily.
4. Wide range of styles and sizes.
5. Off-the-shelf availability.
6. Smaller installation footprint.
7. Less rigorous maintenance requirements.

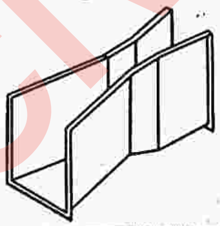


Fig. 2.22.1. Flume.

Que 2.23. **Define venturi flume. Derive the expression for discharge and velocity of flow through a venturi flume.**

Answer

Venturi Flume :

Fig. 2.23.1. Flow through a venturi flume.

1. It is critical flow open flume with constricted flow. This causes drop in hydraulic grade line which creates critical depth.
2. Venturi flume is used for very large flow rate measurements, usually in units of millions of cube.
3. Venturi Flume measures in meters unlike venturimeter which measures in millimeters.
4. Measurement of discharge with venturi flume requires two measurements viz. one at upstream and the other at throat (*i.e.* at narrowest cross-section).

Discharge through Venturi Flume :

1. The width of the channel b_1 is reduced to b_2 to create a throat section. Higher velocity at the throat results in a drop of the depth of the liquid as shown in Fig. 2.23.2.
2. By continuity, $A_1 v_1 = A_2 v_2$
3. By the energy equation, $d_1 + v_1^2/2 = d_2 + v_2^2/2$
4. Head is given by, $h = d_1 - d_2$
5. Discharge, $$Q = \frac{C_d A_1 A_2}{\sqrt{(A_1^2 - A_1^2)}}\sqrt{2gh}$$

Velocity of Flow through Venturi Flume :

We know that, $v = Q/A$

$$v_1 = \frac{Q}{A_1} = \frac{C_d A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh} \Rightarrow v_2 = v_{max} = \frac{Q}{A_2} = \frac{C_d A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

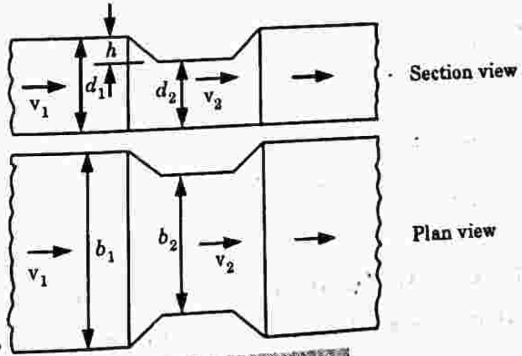


Fig. 2.23.2. Venturi flume.

Que 2.24. **Give the advantages and disadvantages of venturi flume.**

Answer

Advantages of Venturi Flume : Following are the advantages of venturi flume :

1. It has low initial cost.
2. It does not have any potential energy differences compare to weir.
3. Hydraulic head loss is smaller compare to weirs.
4. It has lower pressure drop.
5. It is suitable for unclean waste water also unlike weir.
6. It is very easy to maintain.

Disadvantages of Venturi Flume : Following are the disadvantages of venturi flume :

1. It has nonlinear flow characteristics.
2. When velocity decreases, deposit gets build up and headwater gets obstructed.
3. There is a risk of plugging due to large floating items through it.
4. Measurement is not possible when backflow exists in tail water upto the venturi flume.
5. Quality and reliability of the measurement depend on sensors used in venturi flume.
6. Installation cost is high.
7. It affects local fauna.

Que 2.25. **Define Parshall flume ? What are the uses of Parshall flume ?**

Answer

Parshall Flume : The Parshall flume is a fixed hydraulic structure used to measure the flow of subcritical waters in open channels. Although originally developed to measure irrigation / water rights flow.

Use of Parshall Flume : Following are the various uses of parshall flume :

1. Cooling water discharge.
2. Dam seepage.
3. Industrial effluent.
4. Irrigation / water rights.
5. Landfill leachate.
6. Mine discharge / dewatering.
7. Sanitary sewage (piped and treatment plant).
8. Storm water.

Que 2.26. **Describe the Parshall flume with diagram and give the expression for discharge through Parshall flume.**

Answer

Parshall Flume :

1. The parshall flume is a type of critical depth flume.
2. This flume consists of a converging section with a level floor, a throat with a downstream sloping floor and a diverging section with an adverse slope bed.
3. The discharge in the flume in the free flow mode is given by,

$$Q = KH_a^{\,n} \quad \text{...(2.26.1)}$$

where, K and n = Constants for given flume.

H_a = Heat at a specified location in the converging section.

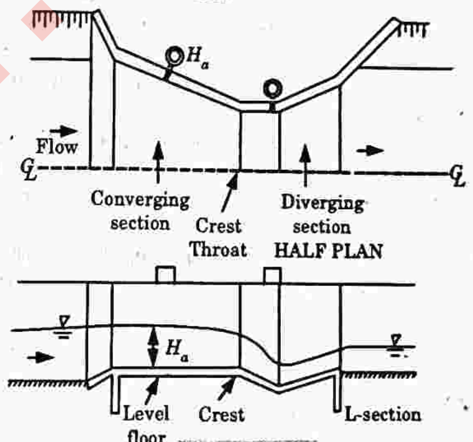


Fig. 2.26.1.

Que 2.27. **Determine the discharge through a Parshall flume having a throat width of 1.5 m for H_a = 1.1 m and H_b = 0.75 m.**

Answer

Given : B_t = 1.5 m, H_a = 1.1 m, H_b = 0.75 m
To Find : Discharge.

If discharge at H_b is less than $0.7H_a$.
0.75 m < (0.7)(1.1 m) = 7.7 m is a true statement, therefore, the discharge through a Parshall flumes is given by,

$$Q = 4.1\, B_t H_a^{1.584}$$

$$Q = 4.1 \times 1.5 \times 1.1^{1.584} = 7.15 \text{ cumecs}$$

Que 2.28. **Define broad-crested weir with neat and clean sketch and also give the expression for discharge through broad crested weir.**

Answer

Broad-Crested Weir :

1. Weirs with a finite crest width in the direction of flow are called broad-crested weirs.
2. They are also termed as weirs with finite crest width and find extensive applications as control structures and flow measuring devices.
3. In this section, the salient flow characteristics of only a simple rectangular, horizontal broad-crested weir are presented.
4. Fig. 2.28.1 is a definition sketch of a free flow over a horizontal broad-crested weir in a rectangular channel.
5. This weir has a sharp upstream corner which causes the flow to separate and then reattach enclosing a separation bubble.
6. If the width B of the weir is sufficiently long, the curvature of the stream lines will be small and the hydrostatic pressure distribution will prevail over most of its width.
7. The weir will act like an inlet with subcritical flow upstream of the weir and supercritical flow over it. A critical-depth control section will occur at the upstream end-probably at a location where the bubble thickness is maximum.
8. Assuming no loss of energy between Sections 1 and 2 (Fig. 2.28.1), and further assuming the depth of flow at Section 2 to be critical,

$$H = y_c + \frac{v_c^2}{2g} = \frac{3}{2} y_c$$

$$v_c = \sqrt{g y_c} \text{ and } y_c = \frac{2}{3} H$$

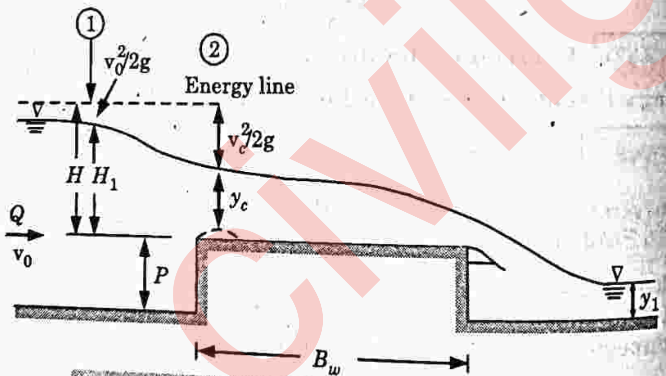


Fig. 2.28.1. Definition sketch of a broad-crested weir.

9. The ideal discharge per unit width of the weir,

$$q_t = v_c y_c = \frac{2}{3}\sqrt{\left(\frac{2}{3} g\right)}\, H^{3/2} = 1.705\, H^{3/2} \quad ...(2.28.1)$$

10. To account for the energy losses and the depth at Section 2 being not strictly equal to the critical depth, the coefficient of discharge C_{d1} is introduced to get an equation for the actual discharge q as

$$q = C_{d1} q_t = 1.705\, C_{d1} H^{3/2} \quad ...(2.28.2)$$

and $Q = qL$

where, L = Length of the weir (transverse direction).

B_w = Width of the weir measured in the longitudinal direction.

11. Since eq. (2.28.2) is rather inconvenient to use as it contains the energy head H, an alternate form of the discharge equation commonly in use is

$$Q = \frac{2}{3} C_d \sqrt{2g}\, L H_1^{3/2} \quad ...(2.28.3)$$

where, H = Height of the water-surface elevation above the weir surface (at upstream).

C_d = Coefficient of discharge.

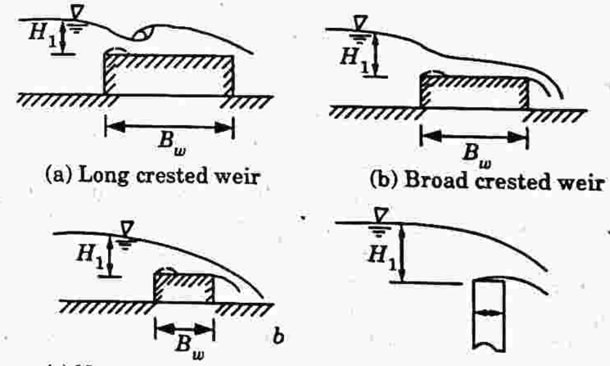
Que 2.29. **Briefly describe the classification of broad-crested weir with diagram.**

Answer

Classification of Broad-Crested Weir :

Based on the value of H_1/B_w the flow over a broad-crested weir with an upstream sharp corner is classified as follows :

1. **$H_1/B_w \le 0.1$:** In this range, the critical flow control section is at the downstream end of the weir and the resistance of the weir surface plays an important role in determining the value of C_d. This kind of weir, termed as long-crested weir, finds limited use as a reliable flow measuring device.



(a) Long crested weir (b) Broad crested weir

(c) Narrow crested weir (b) Sharp crested weir

Fig. 2.29.1.

2. **$0.1 \le H_1/B_w \le 0.35$:** The critical depth control occurs near the upstream end of the weir and the discharge coefficient varies slowly with H_1/B_w in this range. This kind can be called a true broad-crested weir.

3. **$0.35 \le H_1/B_w \le$ about 1.5 :** The water-surface profile will be curvilinear all over the weir. The control section will be at the upstream end. The weirs of this kind can be termed as narrow-crested weirs. The upper limit of this range depends upon the value of H_1/P.
4. **H_1/B_w about 1.5 :** The flow separates at the upstream corner and jumps clear across the weir crest. The flow surface is highly curved, and the weir can be classified as sharp crested.

Que 2.30. **A broad-crested weir with an upstream square corner and spanning the full width of a rectangular canal of 2.0 m width is planned. The proposed crest length is 2.50 m and the crest elevation is 1.20 m above the bed. Calculate the water-surface elevation upstream of the weir when the discharge is (a) 2.0 m³/sec and (b) 3.50 m³/sec.**

Answer

Given : Width of weir = 2 m, length of crest = 2.5 m, height of crest = 1.2 mm, Assume, $C_d = 0.525$
To Find : Elevation of upstream water-surface.

A. For discharge, $Q = 2.0\ \text{m}^3/\text{sec}$
1. Discharge is given by,

$$Q = \frac{2}{3} C_d \sqrt{2g}\, L H_1^{3/2} \qquad \text{...(2.30.1)}$$

$$2.0 = \frac{2}{3} \times 0.525 \times \sqrt{19.62} \times 2.0 \times H_1^{3/2}$$

$$H_1^{3/2} = 0.645 \text{ and } H_1 = 0.747 \text{ m}$$

2. $$\frac{H_1}{B_w} = 0.747/2.5 = 0.299.$$

3. For broad-crested weirs, C_d is given by,

$$C_d = 0.028\,(H_1/B_w) + 0.521$$
$$C_d = 0.028 \times (0.299) + 0.521 = 0.529$$

4. Substituting this C_d value in eq. (2.30.1), we get

$$H_1^{3/2} = 0.640 \Rightarrow H_1 = 0.743 \text{ m and } C_d = 0.529$$

Hence, the water-surface elevation above the bed

$$= 1.2 + 0.743 = 1.943 \text{ m}$$

B. For discharge, $Q = 3.25\ \text{m}^3/\text{sec}$
1. Since Q is higher than in case (A), it is likely that $H_1/B_w > 0.35$. Hence assuming the weir to function in the narrow crested weir mode, the calculations are started by assuming $C_d = 0.55$.
2. From eq. (2.30.1), we get

$$3.50 = \frac{2}{3} \times 0.55 \times \sqrt{19.62} \times 2.0 \times H_1^{3/2}$$

$$H_1^{3/2} = 1.0775 \Rightarrow H_1 = 1.05 \text{ m}$$

3. $$\frac{H_1}{B_w} = \frac{1.05}{2.5} = 0.42$$

The flow is in the transition region between the broad-crested and narrow-crested weir modes.

4. For narrow-crested weirs, C_d is given by,

$$C_d = 0.120\,(H_1/B_w) + 0.492$$
$$C_d = 0.120 \times (0.42) + 0.492 = 0.542$$

5. Put the value of $C_d = 0.542$ in eq. (2.30.1), we get

$$H_1^{3/2} = 1.1 \Rightarrow H_1 = 1.06 \text{ m}$$

6. $$\frac{H_1}{B_w} = \frac{1.06}{2.5} = 0.424$$

7. For narrow-crested weirs, $C_d = 0.120 \times (0.424) + 0.492 = 0.543$
8. From eq. (2.30.1), we get $H_1^{3/2} = 1.0914 \Rightarrow H_1 = 1.060$ m
9. $$\frac{H_1}{B_w} = \frac{1.06}{2.5} = 0.424$$

10. For narrow-crested weirs, $C_d = 0.120\,(0.424) + 0.492 = 0.543$
Hence, $H_1 = 1.060$ and the water-surface elevation above the bed is $1.2 + 1.060 = 2.260$ m

Que 2.31. **Show that for a triangular broad-crested weir flowing free the discharge equation can be expressed as**

$$Q = \frac{16}{25} C_{d1} \tan\theta \sqrt{\frac{2g}{5}}\, H^{5/2}$$

where, H = Energy head measured from the vertex of the weir, θ = Semi-apex angle, and C_{d1} = Coefficient of discharge.

Answer

1. Specific energy for triangular section,

$$H = y_c + \frac{v_c^2}{2g} = \frac{5}{4} y_c \Rightarrow y_c = \frac{4}{5} H$$

2. Area of section, $$A = m y_c^2 = (\tan\theta) \times \frac{16}{25} H^2 \qquad (\because m = \tan\theta)$$

3. Fraud number, $$F = \frac{v\sqrt{2}}{\sqrt{g y_c}}$$

At critical depth, $F = 1$

So, $$1 = \frac{v\sqrt{2}}{\sqrt{g y_c}} \Rightarrow v = \sqrt{\frac{2g}{5}} H^{1/2}$$

4. Discharge is given by, $Q = C_{d1} \times v \times A$

$$Q = C_{d1} \times (\tan\theta) \times \frac{16}{25} H^2 \times \sqrt{\frac{2g}{5}} H^{1/2}$$

$$Q = \frac{16}{25} C_{d1} \tan\theta \sqrt{\frac{2g}{5}}\, H^{1/2}$$

Que 2.32. **Define standing wave flume ? Give the expression for discharge through standing wave flume. Also give its features.**

Answer

Standing Wave Flume :

1. Critical depth flumes are flow-measuring devices in which a control section is achieved through the creation of a critical-flow section by a predominant width constriction.

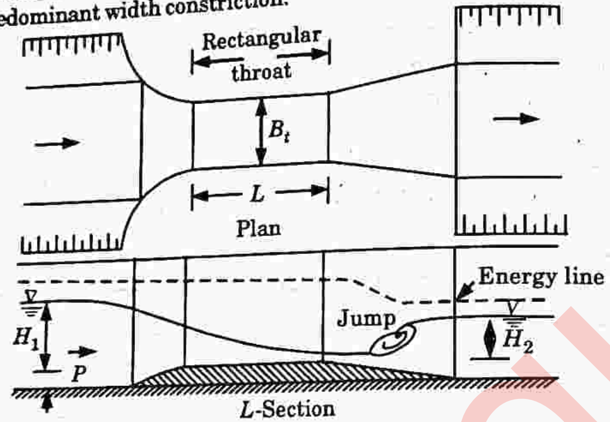


Fig. 2.32.1. Standing-wave flume.

2. A typical critical depth flume consists of a constricted portion called the throat and a diverging section. Sometimes a hump is also provided to assist in the formation of critical flow in the throat.
3. The critical-depth flume shown in Fig. 2.32.1 is known as a standing-wave flume or throated flume. This flume can be fitted into any shape of the parent channel. The throat is prismatic and can be of any convenient shape.
4. A hydraulic jump forms on the downstream of the throat and holds back the tail-water. If the throat is submerged by the tail-water, subcritical flow prevails all over the flume. It is usual to operate the flume in the free-flow mode only, *i.e.*, with the throat unsubmerged.
5. For a rectangular throat section, the discharge is given by,

$$Q = C_f B_t H_1^{3/2} \quad \text{...(2.32.1)}$$

where, C_f = Overall discharge coefficient of the flume $= f(H_1/L)$.

For a well-designed flume, C_f is of the order of 1.62.

H_1 = The difference in the water-surface elevation upstream of the inlet and the elevation of the crest at the throat.

6. **Features of Throated Flumes :**
   i. Low energy loss.
   ii. Rugged construction.
   iii. Easy passage for floating and suspended material load.
   iv. High modular limit.

Que 2.33. **A standing-wave flume without a hump is to be provided in a rectangular channel of bottom width = 2.0 m, n = 0.015 and S_0 = 0.0004. A maximum discharge of 2.50 m³/sec is expected to be passed in this flume. If the modular limit of the flume is 0.75, find the width of the throat. (Assume C_f = 1.62).**

Answer

Given : Bottom width, B = 2 m, Bed slope, S_0 = 0.0004, Discharge, Q = 2.5 m³/sec, Mannings coefficient, n = 0.015, Modular limit = 0.75, C_f = 1.62.

To Find : Width of the throat.

1. $$\phi = \frac{Qn}{\sqrt{S_0}\, B^{8/3}} = \frac{2.5 \times (0.015)}{\sqrt{0.0004}\,(2.0)^{8/3}} = 0.29529$$
2. For ϕ = 0.29529, $\frac{y_0}{B}$ (corresponding to m = 0) = 0.656
3. Normal depth, y_0 = 1.312 m. This is the tail-water depth, H_2.
4. For a modular limit of 0.75, H_1 = 1.312/0.75 = 1.749 m
5. Width of throat, $$B_t = \frac{Q}{C_f H_1^{3/2}} = \frac{2.50}{1.62 \times (1.749)^{3/2}} = 0.667 \text{ m}$$

Que 2.34. **What is current meter ? What are the types of current meter are used in flow measurement ?**

Answer

Current Meter :

1. Current meter is an instrument used for determining the velocity of channel and the rivers. It mainly consist of wheel, contact breaker, tail and the weights.
2. A chart, known as rating chart, is supplied with the current meter, which give relation between the velocity of water and number of revolutions of the wheel per minute.

Classes of Current Meter : Following are the various types of current meter :

1. Electromagnetic current flow meters.
2. Anemometer and propeller current meter.
3. Doppler current meters.
4. Optical strobe current meters.

Que 2.35. **Define electromagnetic flow meter. Explain the working principle of electromagnetic flow meter.**

Answer

Electromagnetic Flow Meter : It is a volumetric flow meter which does not have any moving parts and is ideal for waste-water applications or any dirty liquid which is conductive or water based.

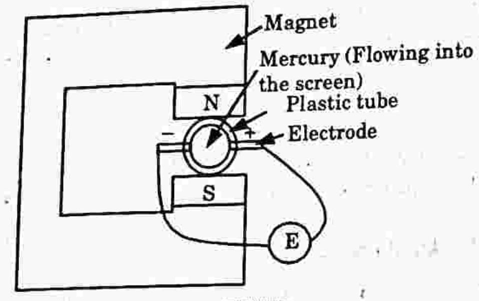


Fig. 2.35.1.

Working Principle :

1. The operation of a electromagnetic flow meter or mag meter is based upon Faraday's law, which states that, "The voltage induced across a conductor as it moves at right angles through the magnetic field is proportional to the velocity of that conductor."
2. According to Faraday's law,

$$E \propto V \times B \times D$$

where,
E = The voltage generated in a conductor.
V = The velocity of the conductor.
B = The magnetic field strength.
D = The length of the conductor.

3. Electromagnetic flow meter use Faraday's law of Electromagnetic Induction to determine the flow of liquid in a pipe.
4. In a magnetic flow meter, a magnetic field is generated and channelled into the liquid flowing through the pipe.
5. Following Faraday's Law, flow of a conductive liquid through the magnetic field will cause a voltage signal to be sensed by electrodes located on the flow tube walls.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. A trapezoidal channel has a bottom width of 6 m and side slopes of 1 : 1. The depth of flow is 1.5 m at a discharge of 15 m³/sec. Determine the specific energy. If the critical depth is 0.9 m, discuss the type of flow corresponding to the critical depth.

Ans. Refer Q. 2.5, Unit-2.

Q. 2. Derive the dynamic equation of GVF, state its various assumptions, also give the limitations of GVF.

Ans. Refer Q. 2.7, Unit-2.

Q. 3. Show that the differential equation of gradually varied flow in a rectangular channel of variable width B can be expressed as

$$\frac{dy}{dx} = \frac{S_0 - S_f + \left(\dfrac{Q^2 y}{gA^3}\dfrac{dB}{dx}\right)}{1 - \dfrac{Q^2 B}{gA^3}}$$

Ans. Refer Q. 2.8, Unit-2.

Q. 4. Integrate the differential equation of GVF for a horizontal channel to get the profile equation as

$$x = \frac{h_c}{S_c}\left[\frac{\left(\dfrac{h}{h_c}\right)^{N-M+1}}{N-M+1} - \frac{\left(\dfrac{h}{h_c}\right)^{N+1}}{N+1}\right] + \text{Constant.}$$

Ans. Refer Q. 2.12, Unit-2.

Q. 5. Classify the various flow profiles with the help of their neat sketches.

Ans. Refer Q. 2.14, Unit-2.

Q. 6. A rectangular channel with a bottom width of 4.0 m and a bottom slope of 0.0008 has a discharge of 1.50 m³/sec. In gradually varies flow in this channel, the depth at certain location is found to be 0.30 m. Assuming n = 0.016, determine the type of GVF profile.

Ans. Refer Q. 2.15, Unit-2.

Q. 7. A rectangular channel 10 m wide is laid with a break in its bottom slope from 0.01 to 0.0064. If it carries 125 m³/sec, determine the nature of the surface profile and compute its length. Take n = 0.015.

Ans. Refer Q. 2.18, Unit-2.

Q. 8. A rectangular channel is 20 m wide and carries discharge of 65 m³/sec. It is laid at a slope of 0.0001. At a certain section along the channel length, the depth of flow is 2.0 m. How far will be the depth be 2.60 m ? Take n = 0.02.

Ans. Refer Q. 2.21, Unit-2.

☺☺☺

3 UNIT

Rapidly Varied Flow and Open Channel Surge

CONTENTS

PART-1

Hydraulic Jump, Evaluation of the Jump Elements in Rectangular and Non-Rectangular Channels on Horizontal and Sloping Beds.

CONCEPT OUTLINE

Hydraulic Jump : The hydraulic jump is defined as the sudden and turbulent passage of water from a supercritical state to subcritical state.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. **What do you understand by term hydraulic jump ? Discuss the classification of hydraulic jump and practical applications of hydraulic jump.** **AKTU 2014-15, Marks 10**

Answer

A. Hydraulic Jump :

1. The hydraulic jump is defined as the sudden and turbulent passage of water from a supercritical state to subcritical state. It has been classified as rapidly varied flow, since the change in depth of flow from rapid to tranquil state is in an abrupt manner over a relatively short distance.
2. The flow in a hydraulic jump is accompanied by the formation of extremely turbulent rollers and there is a considerable dissipation of energy.
3. A hydraulic jump will form when water moving at a supercritical velocity in a relatively shallow stream strikes water having a relatively large depth and subcritical velocity.
4. It occurs frequently in a canal below a regulating sluice, at the foot of a spillway, or at the place where a steep channel bottom slope suddenly changes to a flat slope.

B. Applications of Hydraulic Jump : The applications of hydraulic jump are as follows :

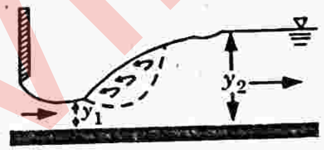
1. It is a useful means of dissipating excess energy of water flowing over spillways and other hydraulic structure or through sluices and thus preventing possible erosion on the downstream side of these structures.
2. It raises the water level in the channels for irrigation.
3. It increases the discharge through a sluice by holding back the tailwater.

4. It increases the weight on an apron of a hydraulic structure due to increased depth of flow and hence the uplift pressure acting on the apron is considerably counterbalanced.

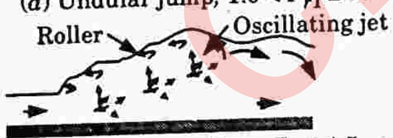
C. **Classification of Hydraulic Jump :** The hydraulic jumps are classified into five categories based on Froude number F_{r1} of the supercritical flow, as follows :

1. **Undular Jump, $1.0 < F_{r1} \le 1.7$:**
 i. The water surface is undulating with a very small ripple on the surface.
 ii. The sequent-depth ratio is very small and E_L / E_1 is practically zero.
2. **Weak Jump, $1.7 < F_{r1} \le 2.5$:** The energy dissipation is very small, *i.e.*, E_L / E_1 is about 5 percent at $F_{r1} = 1.7$ and 18 percent at $F_{r1} = 2.5$. The water surface is smooth after the jump.
3. **Oscillating Jump, $2.5 < F_{r1} \le 4.5$:**
 i. This category of jump is characterized by instability of the high velocity flow in the jump which oscillates in a random manner between the bed and the surface.
 ii. Energy dissipation is moderate in this range, $E_L / E_1 = 45$ percent at $F_{r1} = 4.5$.
4. **Steady Jump, $4.5 < F_{r1} \le 9.0$:**
 i. In this range of Froude numbers, the jump is well established, the roller and jump action is fully developed to cause appreciable energy loss.
 ii. The relative energy loss E_L/E_1 ranges from 45 percent to 70 percent in this class of jump.
5. **Strong or Choppy Jump, $F_{r1} > 9.0$:**
 i. In this class of jump the water surface is very rough and choppy. The water surface downstream of the jump is also rough and wavy.
 ii. The sequent depth ratio is large and the energy dissipation is very efficient with E_L / E_1 values greater than 70 percent.

(*a*) Undular jump, $1.0 < F_{r1} \le 1.7$



(*b*) Weak jump, $1.7 < F_{r1} \le 2.5$



(c) Oscillating jump, $2.5 < F_{r1} \le 4.5$

(*d*) Steady jump, $4.5 < F_{r1} \le 9.0$

(*e*) Strong or Choppy jump, $F_{r1} > 9.0$

Fig. 3.1.1. Classification of jumps.

Que 3.2. **Give the classification of jump on the basis of location of jump.**

Answer

Jump on the basis of location can be classified as follows :

1. **Jumps at Vena Contracta ($y_t = y_2$) :** When $y_t = y_2$, a hydraulic jump will form at vena contracta which is called as free jump at vena contracta.

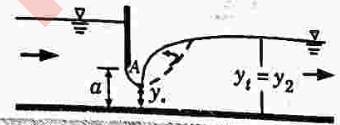


Fig. 3.2.1. Free jump at vena contracta, ($y_t = y_2$).

2. **Repelled Jump ($y_t < y_2$) :** When $y_t < y_2$, the jump is repelled downstream of vena contracta. Such a jump is called repelled jump with sequent depth ($y_{1/2}$) equal or less than y_2.

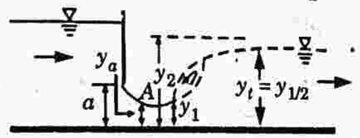


Fig. 3.2.2. Free repelled jump ($y_t < y_2$).

3. **Submerged Jump or Drowned Jump ($y_t > y_2$) :** When $y_t > y_2$, the supercritical stream is submerged and the resulting jump is called submerged or drowned jump.

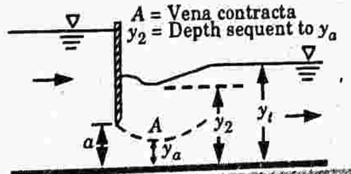


Fig. 3.2.3. Submerged jump, ($y_t < y_2$).

Que 3.3. **Derive the momentum equation formulation for the jump.**

Answer

1. Following assumptions are made in the analysis of hydraulic jump :
 i. The distribution of pressure is hydrostatic.
 ii. Momentum correction factor is unity.
 iii. Loss of head due to friction at the walls and channel bed is negligible.
 iv. The channel is horizontal or it has small slope. The weight component in direction of flow is neglected.

2. Consider a channel is inclined to the horizontal at an angle θ. T channel is assumed to be prismatic channel of arbitrary shape. Secti 1 and 2 refer to the beginning and end of the jump respectively.
3. The flow is considered to be steady. Applying the linear moment equation in the longitudinal direction to the control volume,

$$P_1 - P_2 - F_s + W \sin\theta = M_2 - M_1$$

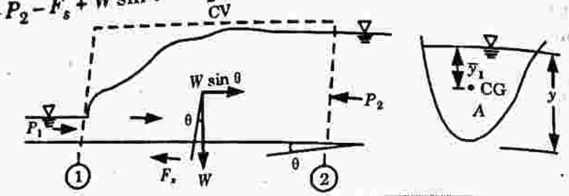


Fig. 3.3.1. Definition sketch for the general momentum equation.

where, P_1 = Pressure force at the control surface at section 1

$= \gamma A_1 \bar{y}_1 \cos\theta$,

$\bar{y}_1$ = Depth of the centroid of the area below the wa surface.

P_2 = Pressure force at the control surface at section 2

$= \gamma A_2 \bar{y}_2 \cos\theta$.

[Note that $P \approx \gamma A\bar{y}$ if θ is small].

F_s = Shear force on the control surface adjacent to t channel boundary.

$W \sin\theta$ = Longitudinal component of the weight of wa contained in the control volume.

M_2 = Momentum flux in the longitudinal direction going through the control surface $= \beta_2 \rho Q v_2$.

M_1 = Momentum flux in the longitudinal direction going through the control surface $= \beta_1 \rho Q v_1$.

5. The hydraulic jump is rapidly-varied flow phenomenon and the len of the jump is relatively small compared of GVF profiles. Thus frictio force F_s is usually neglected as it is of secondary importance.
6. Alternatively for smaller value of Q, $(W \sin\theta - F_s)$ can be considere be very small and hence is neglected. For a horizontal channel, θ and $W \sin\theta = 0$.

Que 3.4. **What is hydraulic jump ? Why momentum equatio used in analysis of hydraulic jump ? Deduce the relation betw alternate depth of hydraulic jump and Froude's number.**

OR

Derive an expression for depth of hydraulic jump in term upstream Froude's number.

AKTU 2018-19, Mark

Answer

A. Hydraulic Jump : Refer Q. 3.1, Page 3–2B, Unit-3.

B. Use of Momentum Equation :

1. Due to high turbulence and shear action of the roller, there is considerable loss of energy in the jump between sections 1 and 2.
2. In view of the high energy loss, the nature of which is difficult to estimate, the energy equation cannot be applied to sections 1 and 2 to relate the various flow parameters.
3. In such situations, we use momentum equation in analysis of hydraulic jump.

C. Expression :

1. Consider a horizontal, frictionless and rectangular channel. Considering unit width of the channel, the momentum equation can be written as

$$\frac{1}{2}\gamma y_1^2 - \frac{1}{2}\gamma y_2^2 = \beta_2 \rho q v_2 - \beta_1 \rho q v_1$$

Taking $\beta_2 = \beta_1 = 1$ and by continuity equation

q = Discharge per unit width $= v_1 y_1 = v_2 y_2$

$$(y_2^2 - y_1^2) = \frac{2q^2}{g}\left(\frac{1}{y_1} - \frac{1}{y_2}\right)$$

i.e., $$y_1 y_2 (y_2 + y_1) = \frac{2q^2}{g} 2y_c^3$$

2. On non-dimensionalizing, $\dfrac{1}{2}\dfrac{y_2}{y_1}\left(1 + \dfrac{y_2}{y_1}\right) = \dfrac{q^2}{g y_1^3} = F_{r1}^2$

where, F_{r1} = Froude number of the approach flow $= \dfrac{v_1}{\sqrt{g y_1}}$

3. Solving for (y_2/y_1) yields, $\dfrac{y_2}{y_1} = \dfrac{1}{2}(1 + \sqrt{1 + 8F_{r1}^2})$
4. This equation which relates the ratio of the sequent depths (y_2/y_1) to the initial Froude number F_{r1} in a horizontal, frictionless, rectangular channel is known as the Belanger momentum equation.
5. For high values of F_{r1}, say $F_{r1} > 8.0$ the sequent depth ratio is,

$$\frac{y_2}{y_1} \approx 1.41 F_{r1}$$

6. Similarly, in terms of F_{r2}, $\dfrac{y_1}{y_2} = \dfrac{1}{2}(-1 + \sqrt{1 + 8F_{r2}^2})$

Que 3.5. **For a hydraulic jump in a horizontal triangular channel show that $3F_{r1}^2 = \dfrac{\gamma^2(\gamma^3 - 1)}{\gamma^2 - 1}$; where, $F_{r1}^2 = \left(\dfrac{v_1^2}{g y_1}\right)$, $\gamma = \left(\dfrac{y_2}{y_1}\right)$**

AKTU 2016-17, Marks 05

Answer

We know that, for triangular channel, $\dfrac{Q^2}{g} = \dfrac{m^2}{3}\dfrac{(y_2^3 - y_1^3)}{\left[\dfrac{y_2^2 - y_1^2}{y_1^2\, y_2^2}\right]}$

$$\frac{A^2 v_1^2}{g} = \frac{m^2}{3}\times\frac{y_1^3[(y_2/y_1)^3-1]\,(y_2/y_1)^2\times y_1^4}{[(y_2/y_1)^2-1]\times y_1^2} \qquad (\because\ Q = Av$$

$$\frac{m^2 y_1^4 v_1^2}{g} = \frac{m^2}{3}\, y_1^5\, \frac{(\gamma^3-1)\,\gamma^2}{(\gamma^2-1)} \qquad (\because\ A = my$$

$$\frac{3v_1^2}{gy_1} = \frac{\gamma^2(\gamma^3-1)}{(\gamma^2-1)} \Rightarrow 3F_{r1}^2 = \frac{\gamma^2(\gamma^3-1)}{(\gamma^2-1)} \qquad \left(\because\ F_{r1}^2 = \frac{v_1^2}{gy_1}\right.$$

Que 3.6. **Derive the equation of energy loss in hydraulic ju for a horizontal rectangular channel.**

Answer

1. The energy loss E_L in the jump is obtained by the energy equati applied to sections 1 and 2 as,

$$E_L = E_1 - E_2 = \left(y_1 + \frac{q^2}{2gy_1^2}\right) - \left(y_2 + \frac{q^2}{2g\,y_2^2}\right) \quad ...(3.6$$

2. Also

$$\frac{q^2}{g} = \frac{y_1^2\, y_2}{2}\left(1 + \frac{y_2}{y_1}\right) \quad ...(3.6$$

3. From eq. (3.6.1) and eq. (3.6.2), we get

$$\therefore \quad E_L = \frac{(y_2 - y_1)^3}{4y_1y_2} \Rightarrow \frac{E_L}{y_1} = \frac{[(y_2/y_1)-1]^3}{4\,(y_2/y_1)}$$

4. The relative energy loss, $\dfrac{E_L}{E_1} = \left(\dfrac{E_L}{y_1}\right)\Big/\left(\dfrac{E_1}{y_1}\right)$ and $\dfrac{E_1}{y_1} = 1 + \dfrac{F_{r1}^2}{2}$

So,
$$E_L/E_1 = \frac{(y_2/y_1 - 1)^3}{4(y_2/y_1)(1 + F_{r1}^2/2)}$$

5. From Belanger momentum equation, $\dfrac{y_2}{y_1} = \dfrac{1}{2}(-1 + \sqrt{1 + 8F_{r1}^2})$

$$\therefore \quad \frac{E_L}{E_1} = \frac{(-3 + \sqrt{1 + 8\,F_r^2})^3}{8\,(2 + F_{r1}^2)(-1 + \sqrt{1 + 8F_{r1}^2})}$$

Que 3.7. **A horizontal rectangular channel of constant width fitted with a sluice gate. When the sluice gate is opened, water issu with a velocity of 6 m/sec and depth of 0.5 m at the vena contra**

Determine whether a hydraulic jump will form or not. If so, calculate the energy dissipated. **AKTU 2014-15, Marks 10**

Answer

Given : Depth before jump, $y_1 = 0.5$ m, Velocity of flow, $v_1 = 6$ m/sec
To Find : Dissipated energy.

1. Froude number decides whether the jump is formed or not. Hence, firstly calculate the Froude number :

$$F_{r1} = \frac{v_1}{\sqrt{gy_1}} = \frac{6}{\sqrt{9.81\times 0.5}} = 2.71$$

Here, the value of F_{r1} varies between 2.5 to 4.5. Hence, the jump is oscillating jump.

2. Sequent depth ratio is given by, $\dfrac{y_2}{y_1} = \dfrac{1}{2}[-1 + \sqrt{1 + 8F_{r1}^{\,2}}]$

$$\frac{y_2}{0.5} = \frac{1}{2}[-1 + \sqrt{1 + 8\times(2.71)^2}] \Rightarrow y_2 = 1.68 \text{ m}$$

3. Dissipated energy, $E_L = \dfrac{(y_2 - y_1)^3}{4y_1y_2} = \dfrac{(1.68 - 0.5)^3}{4\times 0.5\times 1.68} = 0.49$ m

Que 3.8. **A spillway discharge a flood flow at a rate of 7.75 m³/sec per metre width. At the downstream horizontal apron the depth of flow was found to be 0.50 m. What tail water depth is needed to form a hydraulic jump ? If a jump is formed, find its (*a*) type, (*b*) length, (*c*) head loss, (*d*) energy loss as a percentage of the initial energy, and (*e*) profile.** **AKTU 2015-16, Marks 10**

Answer

Given : Discharge, $q = 7.75$ m³/sec/m
Upstream depth of flow, $y_1 = 0.50$ m
To Find : Water depth to form hydraulic jump

1. Velocity of flow at upstream, $v_1 = \dfrac{7.75}{0.50} = 15.50$ m/sec

3. Froude number at upstream, $F_{r1} = \dfrac{15.50}{\sqrt{9.81\times 0.50}} = 7.0$

4. Sequent depth ratio,

$$\frac{y_2}{y_1} = \frac{1}{2}(-1 + \sqrt{1 + 8F_{r1}^2}) = \frac{1}{2}(-1 + \sqrt{1 + 8\times 7^2}) = 9.41$$

5. Required tailwater depth, $y_2 = 0.5 \times 9.41 = 4.71$ m
6. **Type :** Since $F_{r1} = 7.0$, a 'steady' jump will be formed.
7. **Length of Jump :** Since $F_{r1} > 4.5$,

Length of the jump, $L_j = 6.1\, y_2 \Rightarrow L_j = 6.1 \times 4.71 = 28.73$ m

8. **Head Loss :**

Head loss, $E_L = \dfrac{(y_2 - y_1)^3}{4 y_1 y_2} = (4.71 - 0.50)^3/(4 \times 0.5 \times 4.71) = 7.92$ m

9. **Energy Loss :** Energy is given by,

$$E_1 = y_1 + \frac{v_1^2}{2g} = 0.5 + \frac{15.50^2}{2 \times 9.81} = 12.75 \text{ m}$$

$$\frac{E_L}{E_1} = \frac{7.92}{12.75} = 0.621 = 62.1\ \%$$

10. **Profile :**

i. Let us assume, coordinates of the jump profile are (x, h) with the boundary condition.

ii. The jump profile can be expressed in a non-dimensional manner as :

$$\eta = f(\lambda)$$

where,
$$\eta = \frac{h}{0.75\,(y_2 - y_1)} = \frac{h}{0.75 \times (4.71 - 0.5)} = \frac{h}{3.16}$$

and
$$\lambda = x/X$$

We know that, $X/y_1 = 5.08\, F_{r1} - 7.82$

$$X/0.5 = 5.08 \times 7 - 7.82 = 27.74 \Rightarrow X = 13.87$$

$$\lambda = x/13.87$$

iii. The jump profile is obtained in terms of x and h. Noting that the depth $y = h + y_1$, the jump profile $y = f(x)$ is now obtained from the $h = f(x)$ profile.

iv. Table 3.8.1 gives the details of the profile calculation.

Table 3.8.1.

S.No.	λ	η	h(m) $= (\eta \times 3.16)$	x(m) $= (\lambda \times 13.87)$	y(m) $= (h + 0.5)$
1.	0	0	0	0	$y_1 = 0.5$
2.	0.10	0.245	0.77	1.39	1.27
3.	0.30	0.405	1.28	4.16	1.78
4.	0.50	0.570	1.80	6.94	2.30
5.	0.70	0.736	2.33	9.71	2.83
6.	0.90	0.920	2.91	12.48	3.41
7.	1.20	1.105	3.49	16.64	3.99
8.	1.60	1.245	3.93	22.19	4.43
9.	2.069	1.333	4.21	28.70	$y_2 = 4.71$

Note : The value of λ and h are fixed by practical analysis, kindly mug up these values.

Que 3.9. **A horizontal rectangular channel 4 m wide carries a discharge of 16 m³/sec. Determine whether a jump may occur at an initial depth of 0.5 m or not. If a jump occurs determine the sequent depth to this initial depth. Also determine the energy loss in the jump.**

AKTU 2016-17, Marks 05

Answer

Given : Width of channel, $B = 4$ m, Discharge, $Q = 16$ m³/sec
Initial depth, $y_1 = 0.5$ m
To Find : Sequent depth and Energy loss.

1. Velocity in channel, $v = \dfrac{Q}{A} = \dfrac{16}{4 \times 0.5} = 8$ m/sec.

2. Froude number, $F_{r1} = \dfrac{v}{\sqrt{g y_1}} = \dfrac{8}{\sqrt{9.81 \times 0.5}} = 3.61$

Hence the value of Froude number is 3.61, the jump will be formed.

3. Sequent depth ratio,

$$\frac{y_2}{y_1} = \frac{1}{2}[-1 + \sqrt{8F_{r1}^2 + 1}] = \frac{1}{2}[-1 + \sqrt{8 \times (3.61)^2 + 1}]$$

$$y_2/0.5 = 4.63 \Rightarrow y_2 = 2.31 \text{ m}$$

4. The energy loss is given by, $E_L = \dfrac{(y_2 - y_1)^3}{4 y_1 y_2} = \dfrac{(2.31 - 0.5)^3}{4 \times 2.31 \times 0.5} = 1.28$ m

Que 3.10. **Deduce the relations of sequent depths and energy loss in a hydraulic jump occurring in horizontal non-rectangular channel.**

Answer

If the side walls of a channel are not vertical, e.g., in the case of a trapezoidal channel, the flow in a jump will involve lateral expansion of the stream in addition to increase in depth. The cross-sectional areas are not linear functions of the depth of flow. This aspect introduces not only computational difficulties in the calculation of the sequent-depth ratio but also structural changes in the jump.

1. **Basic Equation :**

i. Consider a horizontal frictionless channel of any arbitrary shape. For a hydraulic jump in this channel, the general momentum with the assumption of $\beta_2 = \beta_1 = 1.0$ reduces to

$$P_1 - P_2 = M_2 - M_1 \quad \text{...(3.10.1)}$$

i.e.,
$$\gamma A_1 \bar{y}_1 - \gamma A_2 \bar{y}_2 = \rho Q_2 v_2 - \rho Q_1 v_1 = \frac{\rho Q_2^2}{A_2} - \frac{\rho Q_1^2}{A_1} \quad \text{...(3.10.2)}$$

where A = area of cross-section and $\bar{y}$ = Depth of the centre of gravity of the area from the water surface.

ii. Rearranging eq. (3.10.1), $P_1 + M_1 = P_2 + M_2$

i.e., $$P + M = \gamma\left[A\bar{y} + Q^2/gA\right] = \text{Constant}$$

i.e., $$P + M/\gamma = P_s = A\bar{y} + Q^2/gA = \text{Constant} \quad ...(3.10.3)$$

The term $P_s = \left(\dfrac{P+M}{\gamma}\right)$ is the specific force.

iii. The energy loss E_L due to a jump in a non-rectangular horizontal channel is, $E_L = E_1 - E_2 = (y_1 - y_2) + \dfrac{Q^2}{2g}\left(\dfrac{1}{A_1^2} - \dfrac{1}{A_2^2}\right)$ ...(3.10.4)

2. Sequent-depth Ratio :

i. Expressions for sequent-depth ratios in channels of regular shapes can be obtained by rearranging the term in eq. (3.10.3) to get equality of specific force as

$$\frac{Q^2}{gA_1} + A_1\bar{y}_1 = \frac{Q^2}{gA_2} + A_2\bar{y}_2 \Rightarrow A_2\bar{y}_2 - A_1\bar{y}_1 = \frac{Q^2}{g}\left(\frac{1}{A_1} - \frac{1}{A_2}\right)$$

$$A_1\bar{y}_1\left(\frac{A_2\bar{y}_2}{A_1\bar{y}_1} - 1\right) = \frac{Q^2}{g}\left(\frac{A_2 - A_1}{A_1 A_2}\right)$$

ii. Note that $F_{r1}^2 = \dfrac{Q^2 T_1}{gA_1^3}$, and on re-arranging

$$\left(\frac{A_2}{A_1}\frac{\bar{y}_2}{\bar{y}_1} - 1\right) = F_{r1}^2\left(\frac{A_1/T_1}{\bar{y}_1}\right)\left(1 - \frac{A_1}{A_2}\right) \quad ...(3.10.5)$$

iii. Substituting the expression for A, T and $\bar{y}$ pertinent to the given geometry will lead to an equation relating the sequent-depth ratio to the inlet Froude number and other geometric parameters of the channel.

3. Jumps in Exponential Channels :

i. Exponential channels represent a class of geometric shapes with the area related to the depth as $A = k_1 y^a$ in which k_1 and a are characteristic constants. For example, values of 1.0, 1.5 and 2.0 for a represent rectangular, parabolic and triangular channels respectively.

ii. In the case of an exponential channel (Fig. 3.10.1).

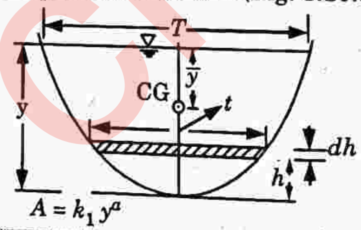


Fig. 3.10.1. Exponential channel.

iii. Top width, $$T = \frac{dA}{dy} = k_1 a\, y^{a-1} \quad ...(3.10.6)$$

$$\frac{A}{T} = (y/a) \Rightarrow \bar{y} = \frac{1}{A}\int_0^y t\,(y-h)\,dh$$

where, t = Top width at any height, $h = \dfrac{1}{A}\displaystyle\int_0^y k_1\, a h^{a-1}(y-h)dh$

iv. Substituting for T, (A/T) and y in eq. (3.10.5), we get

$$\left(\frac{y_2^a}{y_1^a}\frac{y_2}{y_1} - 1\right) = F_{r1}^2\left(\frac{y_1/a}{y_1/(a+1)}\right)\left(1 - \frac{y_1^a}{y_2^a}\right) \quad ...(3.10.7)$$

i.e., $$\left(\frac{y_2}{y_1}\right)^{a+1} - 1 = F_{r1}^2\left(\frac{a+1}{a}\right)\left(1 - \left(\frac{y_1}{y_2}\right)^a\right) \quad ...(3.10.8)$$

Using this equation the ratio y_2/y_1 can be evaluated as a function of F_{r1} and a.

v. The energy loss E_L due to a jump in a horizontal exponential channel can be expressed by using equation (3.10.4) as

$$\frac{E_L}{E_1} = \frac{2a(1 - y_2/y_1) + F_{r1}^2\left[1 - (y_1/y_2)^{2a}\right]}{(2a + F_{r1}^2)} \quad ...(3.10.9)$$

Que 3.11. **In an open channel, the Froude number F remains constant at all depths. If the specific energy E is constant show that**

$$\frac{T}{B} = \left(\frac{E}{E-y}\right)^{\left(\frac{1+F^2}{2}\right)}$$

AKTU 2017-18, Marks 07

Answer

1. Specific energy, $$E = y + \frac{v^2}{2g} = y + \frac{Q^2}{2gA^2} = y + \frac{F^2}{2}\left(\frac{A}{T}\right)$$

$$E - y = \frac{AF^2}{2T}$$

2. Differentiating with respect to y and noting that F is constant,

$$\frac{dE}{dx} - 1 = \frac{F^2}{2}\left[\frac{\left(T\dfrac{dA}{dy} - A\dfrac{dT}{dy}\right)}{T^2}\right]$$

3. Since E is constant, $\dfrac{dE}{dy} = 0$, Also $\dfrac{dA}{dy} = T$

Hence, $$\frac{F^2}{2}\left(1 - \frac{A}{T^2}\frac{dT}{dy}\right) = -1$$

$$\frac{AF^2}{2T}\left(\frac{1}{T}\frac{dT}{dy}\right) = \left(1 + \frac{F^2}{2}\right)$$

4. Substituting for $\left(\frac{AF^2}{2T}\right)$ from eq. (1), $(E-y)\left(\frac{1}{T}\frac{dT}{dy}\right) = \left(1+\frac{F^2}{2}\right)$

$$\frac{dT}{T} = \left(1+\frac{F^2}{2}\right)\frac{dy}{(E-y)}$$

5. On integration, we get

$$\ln T = \left(1+\frac{F^2}{2}\right)[-\ln(E-y)] + C$$

At $y = 0$, $T = B$ and hence $C = \ln B + \left(1+\frac{F^2}{2}\right)\ln E$

$$\therefore \quad \ln\frac{T}{B} = \left(1+\frac{F^2}{2}\right)\ln\left(\frac{E}{E-y}\right)$$

or

$$\frac{T}{B} = \left[\frac{E}{E-y}\right]^{\left[1+\frac{F^2}{2}\right]}$$

Que 3.12. **Evaluate all the salient elements of hydraulic jump on sloping beds.**

Answer

A. Equation :

1. The definition sketch of a jump on a sloping floor in a rectangular frictionless channel is indicated in Fig. 3.12.1. The momentum correction factors β_1 and β_2 are assumed equal to unity.
2. A unit width of the channel is considered with q = Discharge per unit width, y_1 = Depth before the jump and y_t = Depth at the end of the jump.
3. Consider a control volume as shown by dashed lines and the momentum equation in the longitudinal direction would be,

$$P_1 - P_2 + W\sin\theta = M_2 - M_1 \qquad ...(3.12.1)$$

4. Assuming hydrostatic pressure distribution at sections 1 and 2,

$$P_1 = \frac{1}{2}\gamma\, y_1^2 \cos\theta \text{ and } P_2 = \frac{1}{2}\gamma\, y_t^2 \cos\theta$$

5. If the water surface were a straight line joining y_1 and y_t, then the area of the jump $= \frac{1}{2}(y_1 + y_2)\frac{L_j}{\cos\theta}$

(Note that the length of the jump L_j is defined as a horizontal distance between y_1 and y_t).

6. Introducing a coefficient to account for the curvature of the jump profile and $\cos\theta$ term,

$$W = \frac{1}{2}K\gamma L_j(y_1 + y_t) \qquad ...(3.12.2)$$

The momentum flux, $M_1 = \rho q^2/y_1$ and $M_2 = \rho q^2/y_t$

7. Eq. (3.12.1) can now be re-written as

$$\frac{1}{2}\gamma\,[y_1^2\cos\theta - y_t^2\cos\theta + KL_j(y_1+y_t)\sin\theta] = \rho q^2\left(\frac{1}{y_t} - \frac{1}{y_1}\right)$$

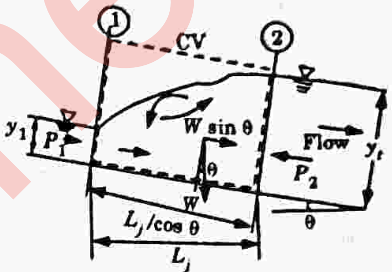


Fig. 3.12.1. Definition sketch for a jump on a sloping floor.

Re-arranging this equation,

$$\left(\frac{y_t}{y_1}\right)^2 - 1 - \frac{KL_j\tan\theta}{y_1}\left(1+\frac{y_t}{y_1}\right) = \frac{2F_{r1}^2}{\cos\theta}\left(\frac{y_t/y_1 - 1}{y_t/y_1}\right)$$

where, $F_{r1} = v_1/\sqrt{gy_1}$.

[Note that F_{r1} is not the exact Froude number of the inclined flow at section $1 = F_{1s}$ but is only a convenient non-dimensional parameter.

8. The Froude number of flow in channels with large θ is given by, for $\alpha = 1.0$, $F_{r1} = v_1/\sqrt{\{g(A/T)\cos\theta\}}$

$$i.e., \left(\frac{y_t}{y_1}\right)^3 - \frac{KL_j\tan\theta}{y_1}\left(\frac{y_t}{y_1}\right)^2 - \left(1+\frac{KL_j\tan\theta}{y_1}+\frac{2F_{r1}^2}{\cos\theta}\right)\left(\frac{y_t}{y_1}\right) + \frac{2F_{r1}^2}{\cos\theta} = 0 \qquad ...(3.12.3)$$

Eq. (3.12.3) can be used to estimate the sequent-depth ratio by a trial-and-error procedure if the term (KL_j) is known.

B. Sequent Depth, y_t :

i. Defining, y_2 = Equivalent depth corresponding to y_1 in a horizontal floor jump $= \frac{y_1}{2}(-1+\sqrt{1+8F_{r1}^2})$, the sequent depth y_t is found to be related to y_2 as $y_t/y_2 = f(\theta)$

2. The variation of (y_t/y_2) with $\tan\theta$ is shown in Fig. 3.12.2. By definition $(y_t/y_2) = 1.0$ when $\tan\theta = 0$ and it is seen from Fig. 3.12.2 that (y_t/y_2) increases with the slope of the channel having typical values of 1.4 and 2.7 at $\tan\theta = 0.10$ and 0.30 respectively.
3. Thus the sloping-floor jumps require more tailwater depths than the corresponding horizontal-floor jumps.

The best fit line for the variation of (y_t/y_2) with $\tan\theta$ shown in Fig. 3.12.2, can be expressed as

$$(y_t/y_2) = 1.0071\exp(3.2386\tan\theta) \qquad ...(3.12.4)$$

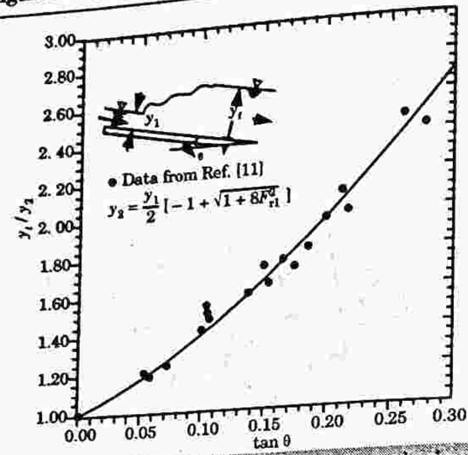


Fig. 3.12.2. Variation of y_t/y_2 in jumps on a sloping floor.

C. Length of the Jump, L_j :

1. The length of the jump L_j is defined as the horizontal distance between the commencement of the jump and a point on the subcritical flow region where the streamlines separate from the floor or to a point on the level water surface immediately downstream of the roller, whichever is longer.
2. The length of the jump on a sloping floor is longer than the corresponding L_j of a jump on a horizontal floor. In the range of $4.0 < F_{r1} < 13$, L_j/y_2 is essentially independent of F_{r1} and is a function of θ only.
3. The variation can be approximately expressed as :

$$L_j/y_2 = 6.1 + 4.0 \tan\theta \quad ...(3.12.5)$$

in the range of $4.5 < F_{r1} < 13.0$.

4. Elevatorski's analysis indicates that the jump length can be expressed as :

$$L_j = m_s(y_t - y_1) \quad ...(3.12.6)$$

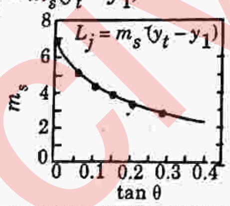


Fig. 3.12.3. Length of jumps on sloping floor.

in which $m_s = f(\theta)$. The variation of m_s with tan θ is shown in Fig. 3.12.3. It may be seen that $m_s = 6.9$ for tan θ = 0 and decreases with an increase in the value of the channel slope.

D. Energy Loss, E_L :

1. Knowing the sequent depths y_t and y_1 and the length of the jump, the energy loss E_L can be calculated as, $E_L = H_1 - H_2$

where, H = Total energy at a section. $E_L = (E_1 + L_j \tan\theta) - E_2$

$$= y_1 \cos\theta + \frac{v_t^2}{2g} + L_j \tan\theta - y_1\cos\theta - \frac{v_1^2}{2g} \quad ...(3.12.7)$$

where, y_t = Sequent depth in a sloping channel at section 2.

2. It is found that the relative energy loss E_L / H_1 decreases with an increase in the value of θ, being highest at tan θ = 0. The absolute value of E_L is a function of θ, being least when θ = 0.

Que 3.13. **A rectangular channel carrying a super critical stream is to be provided with a hydraulic jump type of energy dissipater. It is desired to have an energy loss of 5 m in hydraulic jump when inlet Froude's number is 8.5. What are the sequent depths of this jump ?**

AKTU 2017-18, Marks 07

Answer

Given : Energy loss, E_L = 5 m, Froude's number = 8.5
To Find : Segment depths

1. We know,

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8F_1^2}\right] = \frac{1}{2}\left[-1+\sqrt{1+8\times 8.5^2}\right]$$

$$\frac{y_2}{y_1} = 11.53 \quad ...(3.13.1)$$

2. We know,

$$\frac{E_L}{y_1} = \frac{(y_2/y_1 - 1)^3}{4(y_2/y_1)}$$

$$\frac{5}{y_1} = \frac{(11.53-1)^3}{4\times 11.53} \Rightarrow y_1 = 0.1975 \text{ m}$$

3. From eq. (3.13.1), we get

$$\frac{y_2}{y_1} = 11.53$$

4. Sequent depth, $y_2 = 11.53 \times 0.1975 = 2.28$ m

Que 3.14. **A rectangular channel is laid on a slope of 1 H : 0.15 V. When a discharge of 11 m³/sec/m width is passed down the channel at a depth of 0.7 m, a hydraulic jump is known to occur at a section. Calculate the sequent depth, length of the jump and energy loss in the jump. What would be the energy loss if the slope was zero ?**

Answer

Given : Slope = 1 H : 0.5 V, Discharge, q = 11 m³/sec/m
Depth of flow, y = 0.7 m
To Find : Sequent depth, Length of the jump, Energy loss.

Discharge per unit width, $q = 11$ m³/sec/m

1. At inlet $v_1 = \frac{q}{y_1} = \frac{11}{0.7} = 15.71$ m/sec

2. Froude number, $F_{r1} = \frac{v_1}{\sqrt{gy_1}} = \frac{15.71}{\sqrt{9.81 \times 0.7}} = 6$

3. Equivalent sequent depth in a horizontal floor jump,

$$\frac{y_2}{y_1} = \frac{1}{2}[-1 + \sqrt{1 + 8F_{r1}^2}]$$

$$y_2 = \frac{0.7}{2}[-1 + \sqrt{1 + (8 \times 6^2)}] = 5.6 \text{ m}$$

4. Sequent depth corresponding to a value of $\tan \theta = 0.15$,

$$y_t/y_2 = 1.63$$

$$y_t = \text{Sequent depth in the inclined floor jump}$$
$$= 1.63 \times 5.6 = 9.13 \text{ m}$$

5. **Length of the Jump :**

Corresponding to $\tan \theta = 0.15$, $m_y = 3.8$

$$L_j = m_y (y_t - y_1) = 3.8(9.13 - 0.7) = 32.03 \text{ m}$$

Also $L_j = y_2 (6.1 + 4 \tan \theta)$

$$= 5.6 \times (6.1 + 4 \times 0.15) = 37.52 \text{ m}$$

Average value of $L_j = 34.8$ m

6. **Energy Loss :**

i. Initial specific energy, $E_1 = y_1 \cos \theta + v_1^2/2g$

$$\cos \theta = 0.98893$$

$$E_1 = (0.7 \times 0.98893) + (15.71)^2 / 2 \times 9.81 = 13.278 \text{ m}$$

$$L_j \tan \theta = 34.80 \times 0.15 = 5.22 \text{ m}$$

ii. H_1 = Total energy at section 1 with bed level at 2 as datum

$$= E_1 + L_j \tan \theta = 13.278 + 5.22 = 18.498 \text{ m}$$

iii. $H_2 = E_t = y_t \cos \theta + v_1^2/2g$

$$H_2 = (9.24 \times 0.98893) + \frac{(11/9.24)^2}{2 \times 9.81} = 9.21 \text{ m}$$

iv. Energy loss, $E_L = H_1 - H_2$

i.e., $E_L = (E_1 + L_j \tan \theta) - E_t = 18.498 - 9.21 = 9.288$ m

Also $\frac{E_L}{H_1} \times 100 = \frac{9.288}{18.498} \times 100 = 50.21\%$

7. **For a Horizontal Flow Jump :** If the slope is zero.

i. $y_2 = 5.6$ m, $y_1 = 0.7$ m

ii. $$E_L = \frac{(y_2 - y_1)^3}{4\, y_1 y_2} = \frac{(5.6 - 0.7)^3}{4 \times 0.7 \times 5.6} = 7.503 \text{ m}$$

iii. $$E_1 = y_1 + \frac{v_1^2}{2g} = 0.7 + \frac{15.714^2}{2 \times 9.81} = 13.286 \text{ m}$$

iv. $$\frac{E_L}{E_1} \times 100 = \frac{7.503}{13.286} \times 100 = 56.47\ \%$$

Que 3.15. **Explain the use of the hydraulic jumps as an energy dissipator below a hydraulic structure.**

OR

Hydraulic jump is sometimes used as energy dissipator at the toe of the spillway of a dam. Why ?

AKTU 2015-16, Marks 02

Answer

1. The high energy loss that occurs in a hydraulic jump has led to its adoption as a part of the energy-dissipator system below a hydraulic structure.
2. The downstream portion of the hydraulic structure where the energy dissipation is deliberately allowed to occur so that the outgoing stream can safely be conducted to the channel below is known as a stilling basin.
3. It is a fully-paved channel section and may have additional appurtenances, such as baffle blocks and sills to aid in the efficient performance over a wide range of operating conditions.
4. Stilling basins are so designed that only a good jump with high energy dissipation characteristics is formed within the basin but it is also stable.
5. The simplest type of basin utilizes just the hydraulic jump to dissipate the energy and it should have a minimum length equal to the length of the jump.
6. Hence it converts the kinetic energy into the potential energy of water whereas surge occurs whenever there is a sudden change in the discharge or depth or both.

Que 3.16. **Discuss the different ways for obtaining the hydraulic jump.**

Answer

Conditions for Obtaining the Hydraulic Jump :

1. A hydraulic jump occurs when a supercritical flow (high velocity and small depth) meets a subcritical (low velocity and large depth) flow causing a jump in flow depth.
2. Some examples are given as :

i. At the toe of a spillway, where the high velocity flow over the spillway meets the larger flow depth downstream.

ii. Downstream of a sluice gate, the supercritical flow under the gate meets the uniform flow depth in the downstream channel.

iii. When the bed slope of a channel changes from steep to mild, the supercritical depends on the steep channel jumps to the subcritical depth on the mild channel.

PART-2

Open Channel Surge, Celerity of the Gravity Wave, Deep and Shallow Water Waves, Rectangular Free Overfall.

CONCEPT OUTLINE

Open Channel Surge : A surge or surge wave is a moving wave front which brings about an abrupt change in depth of flow. Surges are usually of two types :

1. Positive surge. 2. Negative surge.

Celerity : The velocity of the surge relative to the initial velocity in the canal is known as the celerity of surge.

For downstream surge, celerity, $C = v_w - v_1$

For upstream surge, celerity, $C = v_w + v_1$

Free Overfall : It is a situation in which there is a sudden drop in the bed causing the flow to separate from the stream bed and move down the step with a free nappe.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.17. **What do you mean by surge ? Give its classification.**

Answer

Surge :

1. A surge or surge wave is a moving wave front which brings about an abrupt change in depth of flow.
2. A surge is also referred to as moving hydraulic jump and is caused by sudden increase or decrease in flow. Flow, such as that caused by sudden opening or closing of a gate fixed in the channel.
3. A surge can travel either in the upstream or downstream direction.

Classification of Surge : Depending upon movement it can be classified into four basic types :

i. Positive surge moving downstream.
ii. Positive surge moving upstream.
iii. Negative surge moving downstream.
iv. Negative surge moving upstream.

4. A positive surge is one in which results in an increase in the depth of flow and negative surges causes a decrease in the depth of flow.

5. Fig. 3.17.1 shows two types of positive surges and two types of negative surges.

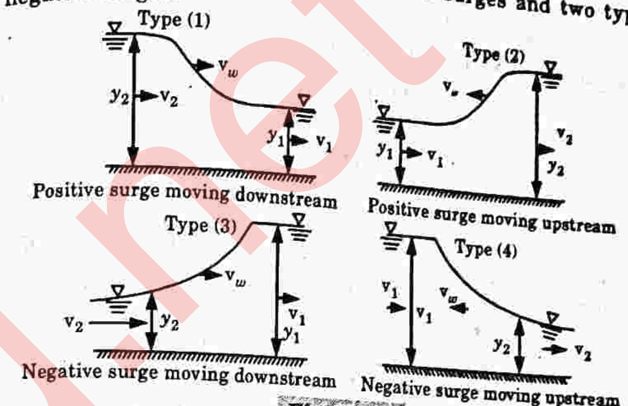


Fig. 3.17.1.

Que 3.18. **Describe positive surge moving upstream and downstream.**

OR

Derive the relation between velocity and depths of flow where positive surges moving upward. **AKTU 2017-18, Marks 07**

Answer

A. Positive Surge Moving Downstream :

1. Consider a sluice gate is suddenly raised to cause positive surge moving downstream. Sections 1 and 2 refer to conditions before and after passage of surge.
2. The absolute velocity v_w is assumed to be constant and let v_1 and v_2 be the velocities and y_1 and y_2 be the corresponding depths at sections 1 and 2 respectively.
3. In order to make it a case of steady flow, apply velocity v_w in opposite directions of to the velocities v_1 and v_2 and the surge. Thus the velocities at sections 1 and 2 becomes $(v_1 - v_w)$ and $(v_2 - v_w)$ respectively.
4. Then from continuity, we have

$$y_2(v_2 - v_w) = y_1(v_1 - v_w)$$

or
$$v_w = \frac{v_1 y_1 - v_2 y_2}{y_1 - y_2} \quad \text{and} \quad v_2 = \frac{y_1(v_1 - v_w) + y_2 v_w}{y_2}$$

5. Further due to the surge developed there is a change in the momentum caused between the sections 1 and 2 and hence the momentum equation may be given as

$$\frac{1}{2}\gamma\, y_2^2 - \frac{1}{2}\gamma\, y_1^2 = \frac{\gamma}{g}\, y_2(v_2 - v_w)\,[(v_1 - v_w) - (v_2 - v_w)]$$

or $$\frac{1}{2}(y_2^2 - y_1^2) = \frac{y_2}{g}(v_2 - v_w)(v_1 - v_2)$$

Substituting the value of v_2, we get

$$(v_w - v_1)^2 = \frac{gy_2}{2y_1}(y_2 + y_1) \text{ or } v_w = \left[\sqrt{\frac{gy_2}{2y_1}(y_1 + y_2)}\right] + v_1$$

Substituting the value of v_w, we get

$$(v_1 - v_2)^2 = (y_1 - y_2)^2 \frac{(y_1 + y_2)g}{2y_1 y_2}$$

This equation shows the relationship among the initial and final velocities and depth of the surge.

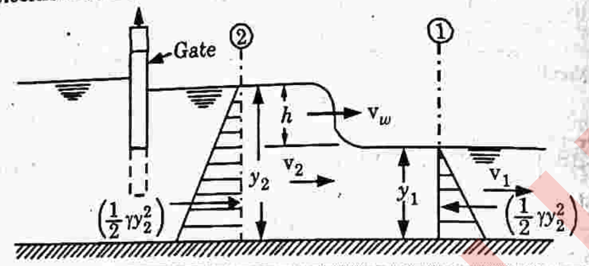


Fig. 3.18.1. Positive surge of type A in a rectangular channel.

B. Positive Surge Moving Upstream :

1. Consider sluice gate is suddenly closed to cause the positive surge moving upstream. Sections 1 and 2 refer to condition before and after passage of surge.
2. The unsteady flow is converted to equivalent steady flow by using relative velocity concept with absolute velocity of surge v_w.
3. Consider unit width of horizontal frictionless rectangular channel. Applying continuity equation, we get

$$y_1(v_w + v_1) = y_2(v_w + v_2) \Rightarrow v_2 = \frac{y_1 v_1}{y_2} - \left(1 - \frac{y_1}{y_2}\right)v_w$$

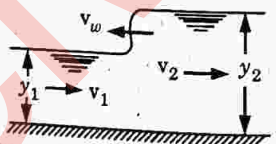


Fig. 3.18.2. Type B : Positive surge (Advancing upstream) ($v_w = C - v_1$)

4. Applying momentum equation, we get

$$\frac{1}{2}\gamma y_1^2 - \frac{1}{2}\gamma y_2^2 = \frac{\gamma}{g} y_1(v_w + v_1)\,[(v_w + v_2) - (v_w + v_1)]$$

$$y_1^2 - y_2^2 = \frac{2y_1}{g}(v_w + v_1)(v_2 - v_1)$$

5. Substituting the value of v_2, we get

$$\frac{(v_w + v_1)^2}{g\,y_1} = \frac{1}{2}\frac{y_2}{y_1}\left(\frac{y_2}{y_1} + 1\right)$$

Que 3.19. **Derive the expression for the following :**

A. Negative surge moving downstream.
B. Negative surge moving upstream.

Answer

A. Negative Surge Moving Downstream :

1. Consider the negative wave moving downstream in a wide rectangular channel due to partially closure of sluice gate.
2. Let initial velocity and depth be v_1 and y_1 and final be v_2 and y_2. The velocity v and depth y at any position x from gate is obtained by integrating differential equation

$$\frac{dv}{dy} = \sqrt{\frac{g}{y}}$$

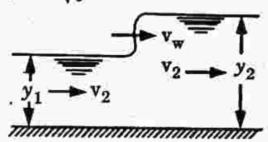


Fig. 3.19.1. Type C : Negative surge (Retreating downstream) ($v_w = C + v_1$).

3. On integration, $v = 2\sqrt{gy} + \text{Constant}$ ...(3.19.1)

   Applying boundary condition, $v = v_1, y = y_1$

   we get $v_1 = 2\sqrt{gy_1} + \text{Constant}$ or $\text{Constant} = v_1 - 2\sqrt{gy_1}$

   $\therefore$ $v = 2\sqrt{gy} - 2\sqrt{gy_1} + v_1$
4. Wave travel downstream, $C = v_w - v = \sqrt{gy}$

   $v_w = v + \sqrt{gy}$ ...(3.19.2)
5. Putting the value of v in eq. (3.19.2), we get

   $v_w = 3\sqrt{gy} - 2\sqrt{gy_1} + v_1$
6. If surge v_w move x distance in time 't' then,

   $x = v_w t$ ...(3.19.3)
7. Putting the value of v_w in eq. (3.19.3), we get

   $x = v_1 + 3\sqrt{gy} - 2\sqrt{gy_1}$

B. Negative Surge Moving Upstream :

1. Consider negative wave produced by instantaneously raising a sluice gate located at downstream end of channel. The basic differential equation of negative wave is

$$\frac{dv}{dy} = -\sqrt{\frac{g}{y}}$$

On integration, $v = -2\sqrt{gy}$ + Constant ...(3.19.4)

2. Let v_1 and y_1 be velocity and depth before passage of wave, v_2 and y_2 after the passage of wave. Applying boundary condition,

at $v = v_1$, $y = y_1$ $\quad v_1 = -2\sqrt{gy_1}$ + Constant

or $\quad$ Constant $= v_1 + 2\sqrt{gy_1}$

Put in eq. (3.19.4) we get

$$v = -2\sqrt{gy} + 2\sqrt{gy_1}\ v_1$$

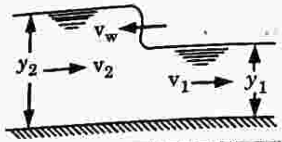


Fig. 3.19.2. Type *D* : Negative surge (Retreating upstream) $(v_w = C - v)$.

3. As negative wave moving upstream, $C = v_w + v = \sqrt{gy}$

$$v_w = \sqrt{gy} - v \quad ...(3.19.5)$$

4. Substituting the value of v in eq. (3.19.5), we get

$$v_w = 3\sqrt{gy} - 2\sqrt{gy_1} - v_1$$

5. Let us consider surge is moving with velocity v_w and moves x distance in time t. Then, $\quad x = -v_w t$ [negative sign shows upstream direction] ...(3.19.6)

6. Substituting the value of v_w in eq. (3.19.6), we get

$$-x = (v_w t) = (3\sqrt{gy} - 2\sqrt{gy_1} - v_1)t$$

Que 3.20. What are back water and celerity ? Prove that celerity $C = \sqrt{gy_1}$ where symbols have usual meanings.

Answer

A. **Back water :** Back water refers to the water backed up in its course by an obstruction (or, sometimes, by an opposing current).

B. **Celerity :**

1. The velocity of the surge relative to the initial velocity in the canal is known as the celerity of the surge.
2. Thus for the surge moving downstream, $C = v_w - v_1$ and for the surge moving upstream, $C = v_w + v_1$.
3. For both the cases, $C = \sqrt{\dfrac{gy_2}{2y_1}(y_2 + y_1)}$ $\quad C = \sqrt{\dfrac{g(y_1 + h)(y_1 + h + y_1)}{2y_1}}$

[where, h = height of surge $= y_2 - y_1$]

$$C = \sqrt{gy_1}\sqrt{1 + \frac{3}{2}\left(\frac{h}{y_1}\right) + \frac{1}{2}\left(\frac{h}{y_1}\right)^2}$$

4. If h is less than y_1, the term $\frac{1}{2}\left(\frac{h}{y_1}\right)^2$ becomes much small and hence it can be neglected.

$$\therefore \quad C = \sqrt{gy_1}\sqrt{1 + \frac{3}{2}\left(\frac{h}{y_1}\right)} \Rightarrow C = \sqrt{gy_1}\left(1 + \frac{3}{4}\frac{h}{y_1}\right)$$

5. Again when the ratio $\left(\frac{h}{y_1}\right)$ is very small, the term $\left(\frac{3}{4}\frac{h}{y_1}\right)$ can also be neglected and then

$$C = \sqrt{gy_1}$$

If $y_1 \rightarrow y$ Then $\quad C = \sqrt{gy}$

Que 3.21. Explain celerity of the gravity wave, deep and shallow water waves.

Answer

A. **Celerity of the Gravity Wave :**

1. The velocity of the surge relative to the initial velocity of the surge in channel is known as celerity of the gravity wave. It is represented by C_s.
2. For surge moving upstream, $C_s = v_w + v_1$
3. For surge moving downstream, $C_s = v_w - v_1$

$$C_s = \sqrt{\frac{1}{2}g\frac{y_2}{y_1}(y_1 + y_2)}$$

4. If $\quad y_2 = y_1$ then $C_s = \sqrt{gy_2}$

B. **Deep and Shallow Gravity Wave :**

1. **Deep Water Wave :** In deep water wave $d \geq (1/2)\lambda$, particles near the surface move in circular paths, making wind wave combination of longitudinal (back and forward) and transverse (up and down) wave motions.

when, $\quad d > (\lambda/4)$

$$C = \sqrt{\frac{g\lambda}{2\pi}} = \sqrt{\frac{g}{K}}, \text{ longer wave move faster.}$$

$$C_g = \frac{1}{2}\sqrt{\frac{g}{K}} = \frac{1}{2}C, \text{ group speed is half of phase speed.}$$

2. **Shallow Water Wave :** In shallow water wave $d < \lambda/2$ particles near the surface compressed and move in the form of ellipse.

where, $\quad d$ = Water depth, and λ = Wave length.

When, $d << \lambda \quad C = \sqrt{gd}$, phase speed independent of wave length.

$\quad C_g = \sqrt{gd} = C_g$, group speed same as phase speed.

Note : A = At deep water the orbital motion of fluid particles decreases rapidly with increasing depth below the surface.
B = At shallow water wave, movement is elliptical.

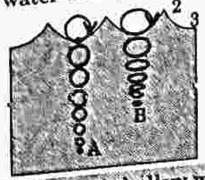


Fig. 3.21.1. Deep and shallow water wave.

Que 3.22. **The depth and velocity of flow in a rectangular channel are 1 m and 1.5 m/sec respectively. If the rate of inflow at the upstream end is suddenly doubled, what will be the height and absolute velocity of the resulting surge and the celerity of the wave ?**

Answer

Given : Depth of flow, $y_1 = 1$ m, Velocity of flow, $v_1 = 1.5$ m/sec
To Find : Height and absolute velocity of surge, celerity of wave.

1. Discharge per unit width, $q_1 = 1 \times 1.5 = 1.5$ m³/sec/m
When discharge is doubled,
$$q_2 = y_2 \times v_2 = 1.5 \times 2 = 3 \text{ m}^3\text{/sec/m}$$
$$y_2 v_2 = 3$$
In this case a positive surge of upstream flow is produced.
2. $$v_w = \frac{v_1 y_1 - v_2 y_2}{y_1 - y_2} = \frac{(1.5 \times 1) - 3}{1 - y_2} = \frac{1.5}{y_2 - 1}$$
3. Also, $$v_w = \left[\sqrt{\frac{g y_2}{2 y_1}(y_2 + y_1)}\right] + v_1$$
$$\frac{1.5}{y_2 - 1} = \left[\sqrt{\frac{9.81 \times y_2}{2 \times 1}(y_2 + 1)}\right] + 1.5$$
Solving this equation, we get $y_2 = 1.284$ m
4. Height of surge, $h = (y_2 - y_1) = 1.284 - 1 = 0.284$ m
5. Absolute velocity of surge, $v_w = 1.5/1.284 - 1 = 5.282$ m/s
6. Celerity of the wave, $C = v_w - v_1 = 5.282 - 1.5 = 3.782$ m/sec

Que 3.23. **Write short note on free overfall through rectangular channel.**

Answer

1. A free overfall is a situation in which there is a sudden drop in the bed causing the flow to separate from the stream bed and move down the step with a free nappe.

2. The situation is analogous to the flow over a sharp-crested weir of zero height. A free overfall causes not only a GVF profile in the subcritical flow, but also offers the possibility of being used as a flow measuring device in all flow regimes.

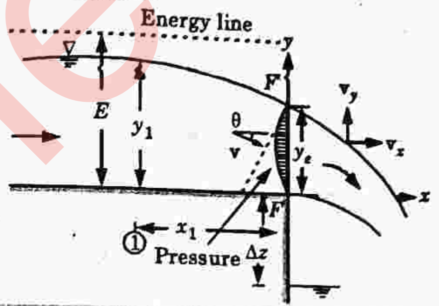


Fig. 3.23.1. Definition sketch of the end depth.

3. A typical free overfall is schematically illustrated in Fig. 3.23.1. The flow in the nappe emerging out of the overfall is obviously affected by gravity.
4. With the atmospheric pressure existing above and below the nappe, the water-surface profile is a parabola.
5. Due to the need for continuity of the water-surface profile, the gravity effect extends a short distance on the water-surface profile behind the edge, causing an acceleration of the flow.
6. Also, at the brink, the pressure should necessarily be atmospheric at points F and F'. This causes the pressure distribution at section FF' to depart from the hydrostatic-pressure distribution and assume a pattern as shown in Fig. 3.23.1.
7. At sections upstream of the brink, the water-surface curvature gradually decreases and at a section such as 1, at a distance x_1 from F, the full hydrostatic pressure is re-established.
8. The result of this effect of the free overfall is to cause a reduction in the depth from Section 1 in the downstream direction with the minimum depth y_e occurring at the brink. This depth y_e is known as the end depth or the brink depth.
9. The critical depth y_c based on hydrostatic pressure distribution will occur upstream of the brink. In Fig. 3.23.1, section 1 can be taken as the critical section with $y_1 = y_c$. Then $x_1 = x_c$. In supercritical flow, y_1 will be equal to the normal depth, $y_1 = y_0$.

Que 3.24. **Prove that the discharge over a spillway is given by the relation, $Q = VD^2 f\left[\sqrt{gD}/V, H/D\right]$** **AKTU 2018-19, Marks 07**

Answer

Note : The relation given is wrong, the correct relation is
$$Q = VD^2 f\left[V/\sqrt{gD}, H/D\right]$$

Solving the equation using above relation.

1. The dimension of each variable are as follows :

Variable	V	D	g	H	Q
Dimension	LT^{-1}	L	LT^{-2}	L	L^3T^{-1}

2. Total number of variable describing the phenomena is, $n = 5$.
3. The number of fundamental dimensions required to describe the variable, $m = 2$.
4. According to Buckingham π theorem, the number of dimensionless π-term $= n - m = 5 - 2 = 3$.
5. Now by using Buckingham π theorem the problem can be expressed as :

$$f(\pi_1, \pi_2, \pi_3) = 0 \quad ...(3.24.1)$$

6. Using D and g as repeating variables, π-terms can be written as

$$\pi_1 = D^{a_1} g^{b_1} V, \ \pi_2 = D^{a_2} g^{b_2} H, \ \pi_3 = D^{a_3} g^{b_3} Q$$

7. **First π-term :** $\pi_1 = D^{a_1} g^{b_1} V \quad ...(3.24.2)$

The variable in eq. (3.24.2) can be expressed in term of their fundamental dimensions.

$$L^oT^o = [L]^{a_1} [LT^{-2}]^{b_1} [LT^{-1}]$$

$$a_1 + b_1 + 1 = 0 \quad \Big| \quad -2b_1 - 1 = 0$$

$$a_1 + b_1 = -1 \quad \Big| \quad b_1 = -1/2$$

then, $a_1 = -1 + 1/2 = -1/2$

Substituting these values in eq. (2), we get

$$\pi_1 = V / \sqrt{gD}$$

8. **Second π-term :** $\pi_2 = D^{a_2} g^{b_2} H \quad ...(3.24.3)$

The variable in eq. (3.24.3) can be expressed in term of their fundamental dimensions.

$$L^oT^o = [L]^{a_2} [LT^{-2}]^{b_2} [L]^1$$

$$-2b_2 = 0 \quad \Big| \quad a_2 + b_2 + 1 = 0$$

$$b_2 = 0 \quad \Big| \quad a_2 = -1$$

Substituting the above values in eq. (3.24.3), we get

$$\pi_2 = H / D$$

9. **Third π-term :**

$$\pi_3 = D^{a_3} g^{b_3} Q \quad ...(3.24.4)$$

The variable in eq. (3.24.4) can be expressed in term of their fundamental dimension.

$$L^oT^o = [L]^{a_3} [LT^{-2}]^{b_3} [L^3T^{-1}]$$

$$-2b_3 - 1 = 0 \quad \Big| \quad a_3 + b_3 + 3 = 0$$

$$b_3 = -1/2 \quad \Big| \quad a_3 = -3 + 1/2 = -5/2$$

Substituting the above values in eq. (3.24.4), we get

$$\pi_3 = Q / (D^{3/2} \times g^{1/2})$$

$$= Q\pi_1 / (\pi_1 D^2 \sqrt{gD}) \text{ (Multiple and divide term-}\pi_1)$$

$$\pi_3 = Q\pi_1 / VD^2 \qquad [\because \pi_1 = V / \sqrt{gD}]$$

10. Value of π_1, π_2 and π_3 putting in eq. (3.24.1), we get

$$f\left(V / \sqrt{gD}, H / D, Q\pi_1 / VD^2\right) = 0$$

$$Q\pi_1 / VD^2 = f\left(V / \sqrt{gD}, H / D\right) \qquad [\because \pi_1 = \text{Constant}]$$

Then we get, $Q = VD^2 f\left[V / \sqrt{gD}, H / D\right]$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. What do you understand by term hydraulic jump ? Discuss the classification of hydraulic jump and practical applications of hydraulic jump.

Ans. Refer Q. 3.1, Unit-3.

Q. 2. Derive the momentum equation for hydraulic the jump.

Ans. Refer Q. 3.3, Unit-3.

Q. 3. A spillway discharge a flood flow at a rate of 7.75 m³/sec per metre width. At the downstream horizontal apron the depth of flow was found to be 0.50 m. What tail water depth is needed to form a hydraulic jump ? If a jump is formed, find its (*a*) type, (*b*) length, (*c*) head loss, (*d*) energy loss as a percentage of the initial energy, and (*e*) profile.

Ans. Refer Q. 3.8, Unit-3.

Q. 4. A horizontal rectangular channel 4 m wide carries a discharge of 16 m³/sec. Determine whether a jump may occur at an initial depth of 0.5 m or not. If a jump occurs determine the sequent depth to this initial depth. Also determine the energy loss in the jump.

Ans. Refer Q. 3.9, Unit-3.

Q. 5. Explain the use of the hydraulic jumps as an energy dissipator below a hydraulic structure.

Ans. Refer Q. 3.15, Unit-3.

Q. 6. What do you understand by surge ? Give the classification of surge.

Ans. Refer Q. 3.17, Unit-3.

Q. 7. Derive the relation between velocity and depths of flow where positive surges moving upward.

Ans. Refer Q. 3.18, Unit-3.

☺☺☺

4 UNIT

Pumps

CONTENTS

PART-1

Impulse Momentum Equation, Impact of Jets Plane and Curved-Stationary and Moving Plates.

CONCEPT OUTLINE

Impact of Jet : It is the force exerted by the jet on a plate which may be stationary or moving.

Formula of Dynamic Force Exerted by Fluid Jet on Stationary Plates and Moving Plates :

Condition	*Stationary*	*Moving*
1. When plate is normal to jet	$F_x = \rho a v^2$	$F_x = \rho a(v-u)^2$
2. Flat plat inclined to jet	$F_x = \rho a v^2 \sin^2\theta$ $F_y = \rho a v^2 \sin\theta\cos\theta$	$F_x = \rho a(v-u)^2 \sin^2\theta$ $F_y = \rho a(v-u)^2 \sin\theta\cos\theta$
3. When plate is curved and jet impinges at the centre of plate.	$F_x = \rho a v^2 (1+\cos\theta)$ $F_y = -\rho a v^2 \sin\theta$	$F_x = \rho a(v-u)^2(1+\cos\theta)$ $F_y = \rho a(v-u)^2(-\sin\theta)$
4. When plate is curved and jet impinges at one end	$F_x = \rho a v^2(\cos\theta + \cos\phi)$ $F_y = \rho a v^2(\sin\theta - \sin\phi)$	$F_x = \rho a v_{r1}(v_{w1} + v_{w2})$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. **Derive linear momentum and impulse momentum equation.**

Answer

1. The fundamental principle of dynamics is the Newton's second law of motion which states that "the rate of change of momentum is proportional to applied force and takes place in the direction of the force."
2. Momentum is the product of mass and velocity. Let m be the mass of fluid moving with velocity v and let the change in velocity be dv in time dt.
3. Change in momentum = mdv

   Rate of change of momentum = $\dfrac{md\text{v}}{dt}$
4. According to Newton's second law,

Dynamic force applied in X-direction = Rate of change of momentum in X-direction

$$F_x = \frac{mdv_x}{dt} \quad ...(4.1.1)$$

5. This equation is known as linear momentum equation. Eq. (4.1.1) can also be written as : ...(4.1.2)

$$F_x\, dt = mdv_x$$

Product of $(F_x \times dt)$ is impulse of applied force.

6. This equation is known as impulse momentum equation. According to this equation,

Impulse of dynamic forces = Resulting change in momentum of body.

Que 4.2. **Define the following with formula (a) Kinetic energy correction factor (b) Momentum correction factor.**

AKTU 2017-18, Marks 07

Answer

A. Kinetic Energy Correction Factor :

1. The kinetic energy correction factor at any section is defined as the ratio of kinetic energy of flow based on actual velocity to the kinetic energy of flow based on average velocity and is found as follows :

$$\alpha \times \rho A v_{av} \times \frac{v_{av}^2}{2} = \frac{1}{2}\int_A \rho v^3\, dA$$

$$\alpha = \frac{\int_A \rho v^3\, dA}{\rho A v_{av}^3} \quad ...(4.2.1)$$

2. For a constant density flow, eq. (4.2.1) reduces to

$$\alpha = \frac{\int_A v^3\, dA}{A v_{av}^3} \quad ...(4.2.2)$$

B. Momentum Correction Factor :

1. The momentum correction factor at any section is defined as the ratio of momentum of flow based on actual velocity to the momentum of flow based on average velocity and is obtained as follows

$$\beta \times \rho A v_{av} \times v_{av} = \int_A \rho v^2\, dA$$

$$\beta = \frac{\int_A \rho v^2\, dA}{\rho A v_{av}^2} \quad ...(4.2.3)$$

2. For a constant density flow, eq. (4.2.3) reduces to

$$\beta = \frac{\int_A v^2\, dA}{A v_{av}^2} \quad ...(4.2.4)$$

Que 4.3. **Derive the formula for dynamic force exerted by fluid jet on stationary plate for the following cases :**

A. When plate is normal to jet.

B. Flat plate inclined to jet.

C. When plate is curved and jet impinges at the center of plate.

D. When plate is unsymmetrical and curved and jet impinges at one end.

Answer

A. When the Plate is Normal to Jet :

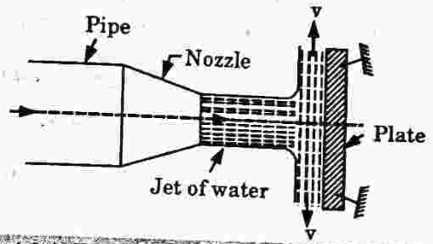


Fig. 4.3.1. Force exerted by jet on vertical plate.

1. Consider a jet of water coming out from the nozzle, strike a flat vertical plate as shown in Fig. 4.3.1.

   v = Velocity of jet. d = Diameter of jet.

   a = Area of cross-section of jet = $\pi/4\, d^2$, ρ = Density of water.

   Q = Discharge of water (m^3/sec).

2. Plate is at 90° to the jet, and jet after striking will move along the plate. So, velocity component of water after strike, in direction of jet will be zero.

3. Dynamic force exerted by the jet on the plate in the direction of jet is calculated as,

F_x = Rate of change of momentum in the direction of force.

= Mass striking the plate/sec × [Change in velocity in direction of jet]

$$= \frac{\text{Mass}}{\text{Time}}\ [\text{Initial velocity} - \text{Final velocity}]$$

$$= \frac{m}{t}[v - 0] = \rho Q\,[v - 0] \qquad \left(\because \left[\frac{m}{t} = \rho Q\right]\right)$$

$$= \rho a v \times v \qquad [\because \text{Discharge } (Q) = av]$$

$$F_x = \rho a v^2$$

B. Flat Plate Inclined to Jet :

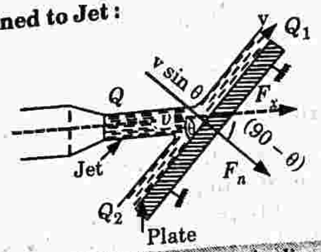


Fig. 4.3.2. Jet striking stationary inclined plate.

1. Here, θ = Angle between the jet and plate.
2. If plate is smooth and there is no loss of energy, then jet will move over the plate with a velocity (v) as shown in Fig. 4.3.2.
3. Now, normal force is calculated as,

F_n = Mass of jet striking the plate/sec × [Change in velocity in normal direction to the plate]

$$= \frac{m}{t}[\text{v} \sin\theta - 0] = \rho Q\,[\text{v} \sin\theta]$$

$$= \rho a\text{v} \times \text{v} \sin\theta = \rho a\text{v}^2 \sin\theta \qquad [\because\ Q = a\text{v}]$$

4. Horizontal component of force,

$$F_x = F_n \sin\theta = \rho a\text{v}^2 \sin\theta \times \sin\theta = \rho a\text{v}^2 \sin^2\theta$$

5. Vertical component of force,

$$F_y = F_n \cos\theta = \rho a\text{v}^2 \sin\theta \times \cos\theta = \rho a\text{v}^2 \sin\theta \cos\theta$$

6. Let discharge along the plate in upward direction is Q_1 and in downward direction is Q_2.
7. So component of Q in downward direction is $Q \cos\theta$.

$$Q\cos\theta + Q_2 = Q_1 \Rightarrow Q\cos\theta = Q_1 - Q_2 \qquad \text{...(4.3.1)}$$

Total discharge is given as, $Q = Q_1 + Q_2$ ...(4.3.2)

8. After solving these two equations, we get

$$Q_1 = \frac{Q}{2}(1 + \cos\theta) \text{ and } Q_2 = \frac{Q}{2}(1 - \cos\theta)$$

C. When Plate is Curved and Jet Impinges at the Center of Plate :

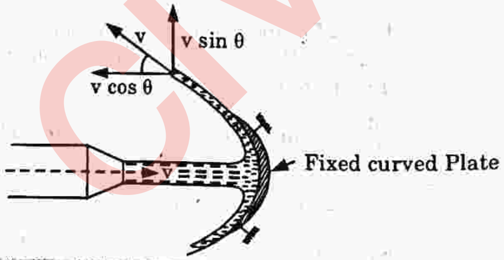


Fig. 4.3.3. Jet striking a fixed curved plate at center.

1. As shown in Fig. 4.3.3.

Component of velocity in direction of jet = – v cos θ.

Component of velocity in perpendicular direction of jet = v sin θ

2. Dynamic force exerted by the jet in X-direction,

F_x = Mass of jet striking the plate/s × [Change in velocity in direction of the plate]

$$= \frac{m}{t}[\text{v} - (-\text{v}\cos\theta)] = \rho Q\,[\text{v} + \text{v}\cos\theta] \qquad \left[\because \frac{m}{t} = \rho Q = \rho a\text{v}\right]$$

$$= \rho a\text{v}\,[\text{v} + \text{v}\cos\theta] = \rho a\text{v}^2(1 + \cos\theta)$$

3. Dynamic force exerted by the jet in direction perpendicular to jet,

F_y = Mass of jet striking/s × [Change in velocity in normal direction to plate]

$$= \frac{m}{t}[0 - (\text{v}\sin\theta)] = \rho Q(-\text{v}\sin\theta) = \rho a\text{v}(-\text{v}\sin\theta)$$

$$F_y = -\rho a\text{v}^2 \sin\theta$$

(Negative sign indicate that force acting in downward direction)

D. When Plate is Unsymmetrical and Curved and Jet Striking at One End :

1. Here, θ = Angle made by jet with X-axis at inlet tip of the curved plate, and

ϕ = Angle made by jet with X-axis at outlet tip of the curved plate.

2. Components of velocity is resolved as at inlet of curved plate,

In X-direction = v cos θ In Y-direction = v sin θ

3. Similarly, at outlet of curved plate,

In X-direction = – v cos ϕ and In Y-direction = v sin ϕ

4. Now, force exerted by jet of water in X-direction,

F_x = Mass of jet striking the plate/s × [Change in velocity in X-direction]

$$= \frac{m}{t}[\text{v}\cos\theta - (-\text{v}\cos\phi)] \qquad \left[\because \frac{m}{t} = \rho Q = \rho a\text{v}\right]$$

$$= \rho Q[\text{v}\cos\theta + \text{v}\cos\phi] = \rho a\text{v}\,(\text{v}\cos\theta + \text{v}\cos\phi)$$

$$F_x = \rho a\text{v}^2(\cos\theta + \cos\phi)$$

4. Force exerted by jet of water in Y-direction,

F_y = Mass of jet striking the plate/s × [Change in velocity in Y-direction]

$$= \frac{\text{Mass}}{\text{Time}} \times [\text{Initial velocity of jet in Y-direction} - \text{Final velocity of jet in Y-direction}]$$

$$= \frac{m}{t}[\text{v}\sin\theta - \text{v}\sin\phi] = \rho a\text{v}\,[\text{v}\sin\theta - \text{v}\sin\phi]$$

$$= \rho a\text{v}^2(\sin\theta - \sin\phi)$$

Note : When the plate is symmetrical, θ = ϕ

So, $F_x = 2\rho a\text{v}^2\cos\theta$, and $F_y = 0$

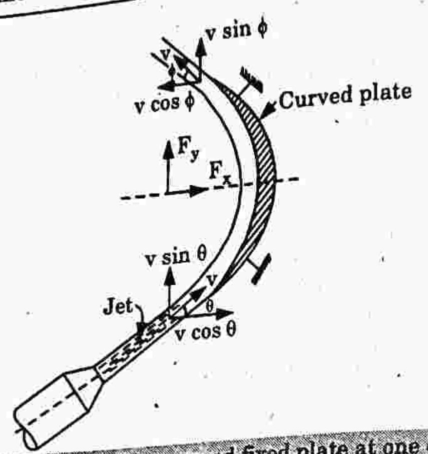


Fig. 4.3.4. Jet striking curved fixed plate at one end.

Que 4.4. **A jet of water of 40 mm diameter strikes a hinged square plate at its centre, with a velocity of 20 m/sec. The plate is deflected through an angle of 30°. Find the weight of the plate. If the plate is not allowed to swing, what will be the force required at the lower edge of the plate to keep the plate in vertical position.**

AKTU 2018-19, Marks 07

Answer

Given : Diameter of the jet, d = 40 mm = 4 cm = 0.04 m
Velocity of jet, v = 20 m/s, Angle of swing, θ = 30°
To Find : Weight of the plate and required force to keep the plate in vertical position.

1. Area of jet, $a = \pi d^2 / 4 = \pi \times 0.04^2 / 4 = 1.2566 \times 10^{-3}\ \text{m}^2$
2. Angle of swing is given by, $\sin \theta = \rho a V^2 / W$
   or $\sin 30° = 1000 \times 1.2566 \times 10^{-3} \times 20^2 / W \Rightarrow W = 1005.28\ \text{N}$
3. If the plate is not allowed to swing, a force P will be applied at the lower edge of the plate as shown in Fig. 4.4.1. The weight of the plate is acting vertically downward through the CG of the plate.

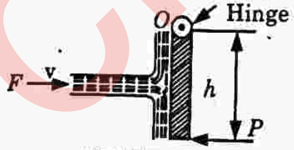


Fig. 4.4.1.

Let, F = Force exerted by jet of water
h = Height of plate = Distance of P from the hinge.

4. The jet strikes at the centre of the plate and hence distance of the centre of the jet from hinge = $h/2$.
5. Taking moments about the hinge, O, $P \times h = F \times h/2$
   ∴ $P = F/2 = \rho a v^2 / 2$ $(\because\ F = \rho a v^2)$
   $= 1000 \times 1.2566 \times 10^{-3} \times 20^2 / 2 = 251.32\ \text{N}$

Que 4.5. **Derive the formula for dynamic force exerted by fluid jet on moving plate for the following cases :**
A. When plate is normal to jet.
B. Flat plate inclined to jet.
C. When plate is curved and jet impinges at the center of plate
D. When plate is curved and jet impinges at one end.

Answer

A. When the Plate is Normal to Jet :

1. Let, v = Velocity of jet,
   a = Area of cross-section of the jet, and
   u = Velocity of the flat plate.
2. Mass of water striking the plate/sec = ρ × Area of jet × Relative velocity
   $= \rho \times a \times (v - u)$ ...(4.5.1)

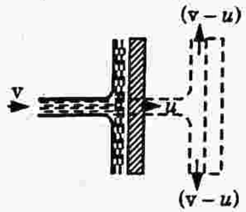


Fig. 4.5.1. Jet striking a flat vertical moving plate.

3. Force exerted by the jet on the moving plate in direction of the jet,
   F_x = Mass of water striking per second × Change in velocity of jet
   $= \dfrac{\text{Mass}}{\text{Time}} \times$ [Initial velocity with which water strike – Final velocity]
   $= \rho a(v - u)\,[(v - u) - 0] = \rho a(v - u)^2$
4. In this case, plate is also moving, so we can calculate the workdone per second,
   $= \text{Force} \times \dfrac{\text{Distance in direction of force}}{\text{Time}}$
   $= \text{Force} \times \text{Velocity of plate} = F_x \times u$
   $\text{Workdone/sec} = \rho a(v - u)^2 \times u$

b. Flat Plate Inclined to the Jet :

1. Here, u = Velocity of plate in direction of jet, and
   θ = Angle between the plate and jet.
2. Relative velocity of jet with respect to plate = $(v - u)$
   [∴ This is the velocity by which jet strike on plate]

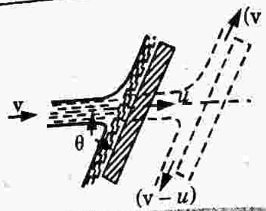


Fig. 4.5.2. Jet striking on inclined moving plate.

3. Mass of water striking the plate per second

$= \rho \times$ Area of jet $\times$ Relative velocity $= \rho \times a \times (v - u)$

4. Component of relative velocity of jet striking normal to the plate

$= (v - u) \sin \theta$

5. Force exerted by the jet in normal direction of plate,

F_n = Mass of water strike per second × Change in velocity normal to plate

= Mass of water strike per second × [Initial velocity in normal direction with which jet strike – Final velocity]

$= \rho a(v - u)\,[(v - u)\sin \theta - 0]$

$F_n = \rho a (v - u)^2 \sin \theta$

6. Force exerted in X-direction by the jet,

$F_x = F_n \sin \theta = \rho a (v - u)^2 \sin \theta \times \sin \theta = \rho a (v - u)^2 \sin^2 \theta$

7. Force exerted in Y-direction by the jet,

$F_y = F_n \cos \theta = \rho a (v - u)^2 \sin \theta \times \cos \theta = \rho a (v - u)^2 \sin \theta \cos \theta$

C. When the Plate is Curved and Jet Striking at the Center of Plate :

1. Relative velocity of jet with respect to plate or velocity of jet with which jet strike on the plate = (v – u).
2. If the plate is smooth and there is no loss of energy due to impact of jet, then the jet will leave the plate with same velocity by which jet strike the plate.

So, velocity of jet leaving the plate = (v – u)

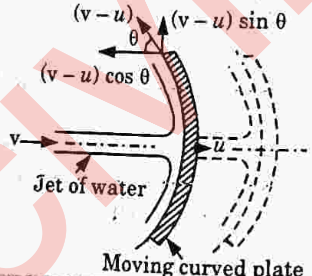


Fig. 4.5.3. Jet striking a curved moving plate.

3. Now, component of velocity in direction of jet = $(v - u) \cos \theta$

Component of velocity perpendicular to the direction of jet = $(v - u) \sin \theta$

4. Mass of water striking the plate per second

$= \rho \times$ Area of jet × Relative velocity by which jet strike plate

$= \rho \times a \times (v - u)$

5. Force exerted by the jet of water on the curved plate in direction of jet,

F_x = Mass of water strike the plate/s × Change in velocity in direction of jet

$= \rho \times a \times (v - u)$ [Initial velocity – Final velocity]

$= \rho \times a \times (v - u)\,[(v - u) - \{-(v - u)\cos \theta\}]$

$= \rho a(v - u)\,[(v - u) + (v - u)\cos \theta] = \rho a (v - u)^2 (1 + \cos \theta)$

6. Force exerted by the jet of water in perpendicular direction of jet,

F_y = Mass of water strike the plate/sec × Change in velocity in perpendicular direction to the jet

$= \rho a (v - u)\,[0 - (v - u)\sin \theta] = -\rho a (v - u)^2 \sin \theta$

7. Workdone by the jet on the plate in direction of jet per second,

= Force (F_x) × Distance travelled per second in direction of X

$= F_x \times u$

$= \rho a (v - u)^2 (1 + \cos \theta) \times u = \rho a (v - u)^2 u (1 + \cos \theta)$

D. When the Plate is Curved and Jet Impinges at One End :

1. As shown in Fig. 4.5.4

At inlet of the plate following terms are represented as,

v_1 (represent by AB) = Velocity of jet at inlet,

u_1 (represent by AC) = Velocity of plate,

v_{r1} (represent by CB) = Relative velocity of jet and plate,

v_{w1} (represent by AD) = Component of velocity of jet v_1 in X-direction also known as velocity of whirl at inlet,

v_{f1} (represent by BD) = Component of velocity of jet v_1 in Y-direction, also known as velocity of flow at inlet,

α = Angle between the direction of jet and direction of motion of plate also known as guide blade angle, and

θ = Angle made by relative velocity (v_{r1}) with direction of motion at inlet, also known as vane angle at inlet.

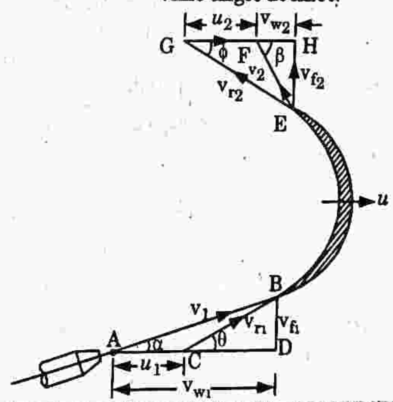


Fig. 4.5.4. Jet striking a moving curved vane at one of the tips.

2. At outlet of the plate, following term are represented as,

v_2 (represent by FE) = Velocity of jet at outlet.

u_2 (represent by GF) = Velocity of plate.

v_{r2} (represent by EG) = Relative velocity of jet leaving the plate.

v_{w2} (represent by FH) = Component of velocity of jet v_2 in X-direction also known as velocity of whirl at outlet.

v_{f2} (represent by EH) = Component of velocity of jet v_2 in Y-direction also known as velocity of flow at outlet.

β = Angle made by velocity of jet v_2, and

ϕ = Angle made by relative velocity v_{r2}.

ΔABD and ΔEGH are called velocity triangles at inlet and outlet.

3. **From Velocity Diagram :**

i. Mass of water strike the plate/sec

= $\rho \times$ Area of jet $\times$ Relative velocity = $\rho \times a \times v_{r1}$

ii. Relative velocity at inlet in X-direction by which jet of water strike = $(v_{w1} - u_1)$

iii. Relative velocity at outlet in X-direction by which jet leaving = $(v_{w2} + u_2)$

iv. Now, force exerted by the jet in the direction of motion,

F_x = Mass of water strike/sec $\times$ Change in velocity in X-direction ...(4.5.1)

$F_x = \rho \times a \times v_{r1} \times$ [Relative velocity at inlet in X-direction – Relative velocity at outlet in X-direction]

$= \rho \times a \times v_{r1} \times [(v_{w1} - u_1) - \{-(u_2 + v_{w2})\}]$ [$\because u_1 = u_2$]

$F_x = \rho a\, v_{r1}\,(v_{w1} + v_{w2})$ ...(4.5.2)

4. **Special Cases :**

i. If at outlet $\beta > 90°$, then exerted force is given as, $F_x = \rho a v_{r1}\,[v_{w1} - v_{w2}]$

ii. If at outlet $\beta = 0°$, then $v_{w2} = 0$

5. Exerted force is given as, $F_x = \rho a v_{r1} \times v_{w1}$

So, in general, we can write the expression for force as,

$F_x = \rho a v_{r1} [v_{w1} \pm v_{w2}]$

6. Now, workdone per second on the vane by the jet is given as,

Workdone = Force $\times$ Distance per second in direction of force

$= F_x \times u = \rho a v_{r1}\,[v_{w1} \pm v_{w2}] \times u$ ...(4.5.3)

[Here $u = u_1 = u_2$]

7. Workdone/sec per unit mass of fluid striking the plate per second,

$$= \frac{\rho a v_{r1}[v_{w1} \pm v_{w2}] \times u}{\text{Mass of fluid striking/sec}} = \frac{\rho a v_{r1}[v_{w1} \pm v_{w2}] \times u}{\rho a v_{r1}} \frac{\text{Nm/sec}}{\text{kg/sec}}$$

$$= (v_{w1} \pm v_{w2}) \times u \;\; \text{Nm/kg} \qquad ...(4.5.4)$$

8. Workdone/sec per unit weight of fluid striking the plate per second,

$$= \frac{\rho a v_{r1}[v_{w1} \pm v_{w2}] \times u}{\text{Weight of fluid/sec}} = \frac{\rho a v_{r1}[v_{w1} \pm v_{w2}] \times u}{(\text{Mass of fluid} \times g)/\text{sec}}$$

$$= \frac{\rho a v_{r1}[v_{w1} \pm v_{w2}] \times u}{\rho a v_{r1} \times g} = \frac{(v_{w1} \pm v_{w2})\, u}{g} \;\; \text{Nm/N} \qquad ...(4.5.5)$$

Que 4.6. **A jet of water from a fixed nozzle has a diameter of 25 mm and strikes a flat plate at an angle of 30° to the normal to the plate. The velocity of jet is 5 m/sec and surface of plate is frictionless. Calculate the force exerted normal to the plate :**

i. If the plate is stationary, and

ii. If the plate is moving with a velocity u of 2 m/sec in the same direction as the jet.

Answer

Given : Jet diameter, d = 25 mm, Angle of plate, $\theta = 30°$

Velocity of jet, v = 5 m/sec

To Find : Force exerted to the plate.

Cross section are of jet, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.025)^2 = 0.00049\ \text{m}^2$

i. When plate is stationary, Normal force, $F_n = \rho a v^2 \sin\theta$

$= 1000 \times 0.00049 \times (5)^2 \times \sin 30° = 6.125$ N

ii. When plate is moving with u = 2 m/sec

Normal force, $F_n = \rho a (v - u)^2 \sin\theta$

$= 1000 \times 0.00049 \times (5-2)^2 \times \sin 30° = 2.205$ N

PART-2

Positive Displacement Pumps, Reciprocating Pumps, Operating Principles, Slip, Indicator Diagram, Separation, Air Vessels.

CONCEPT OUTLINE

Pump : It may be defined as a mechanical device which when interposed in a pipeline, converts the mechanical energy supplied to it from some external source into hydraulic energy and transfer the same to the liquid through the pipeline, thereby increasing the energy of the flowing liquid.

Types :

1. Positive-displacement pumps.
2. Rotodynamic pumps (or dynamic-pressure pumps).

Reciprocating Pump : If the mechanical energy is converted into hydraulic energy by sucking the liquid into a cylinder in which a piston is reciprocating, which exerts the thrust on the liquid and increase its hydraulic energy, the pump is known as reciprocating pump.

Slip of a Reciprocating Pump :

It is defined as the difference between the theoretical discharge and actual discharge of the pump.

$$\text{Slip} = Q_{th} - Q_{act}$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.7. **What is reciprocating pump ? Describe the principle and working of a reciprocating pump with a neat sketch.**

AKTU 2018-19, Marks 07

OR

Explain the working principles of reciprocating pump.

AKTU 2016-17, Marks 02

Answer

A. **Reciprocating pump :** If mechanical energy is converted into pressure energy by means of reciprocating motion of a piston into a cylinder, then pump is known as reciprocating pump.

B. **Components :** Main parts of a reciprocating pump are :

1. Cylinder with piston, piston rod, connecting rod and a crank.
2. Suction pipe.
3. Delivery pipe.
4. Suction valve.
5. Delivery valve.

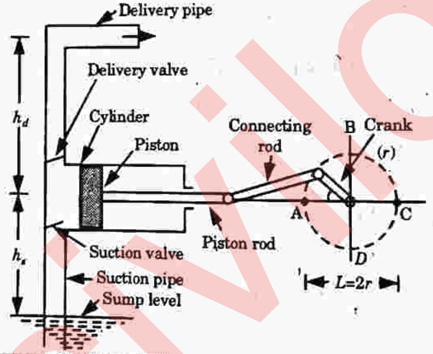


Fig. 4.7.1. Main parts of a reciprocating pump.

C. **Working Principle :**

1. A reciprocating pump consists of a piston or a plunger executing reciprocating motion inside a cylinder.
2. As the crank moves outwards (from A to C), the piston moves towards right in the cylinder causing a vacuum in the cylinder.
3. Due to the pressure difference between the sump (atmospheric pressure) and the cylinder, liquid is drawn into the cylinder through the non-return suction (or inlet) value.
4. During this outward stroke, the delivery valve remains closed.

5. During the return stroke of the crank (from C to A), the piston moves towards the left causing an increase in pressure in the cylinder which opens the delivery valve and closes the inlet valve.
6. The liquid is forced into the delivery pipe and is raised to a required height.

Que 4.8. **Differentiate between single acting and Double acting reciprocating pump.**

Answer

S. No.	Single Acting Pump	Double Acting Pump
1.	The liquid being pumped is in contact with one side of piston/plunger of pump.	The liquid being pumped is in contact with both side of piston/ plunger of pump.
2.	It has only one delivery stroke for one complete revolution of the crank.	It has two delivery stroke for one complete revolution of the crank.
3.	Discharge is less.	Discharge is more.
4.	Power required is less.	Power required is more.
5.	Work saved by fitting air vessel is 84.8 %.	Work saved by fitting air vessel is 39.2 %.

Que 4.9. **Differentiate between single stage and multistage pumps.**

AKTU 2018-19, Marks 07

Answer

Differentiate between single stage and multistage pumps :

S. No.	Single Stage Pump	Multi Stage Pump
1.	Single stage pump is the only one impeller pump.	Multi stage pump refers to two or more than two of the pump impeller.
2.	The maximum lift is only 125 meter.	The maximum lift more than 125 meter.
3.	It have Low cost.	It have high cost.
4.	In case of single stage pump head needs to be quipped with two motors.	In case of of multi stage Pump. Can increase the number of impeller to equipped with four motors, which can improve the life of the pump and reduce unit noise.

Que 4.10. **Define indicator diagram and also explain the ideal indicator diagram.**

Answer

A. Indicator Diagram :

1. It is defined as the graph between the pressure head in the cylinder and distance travelled by piston from inner dead center for one complete revolution of crank.
2. In reciprocating pump, maximum distance travelled by piston is stroke length. So indicator diagram is a graph between pressure head and stroke length.
3. Pressure head is taken as ordinate and stroke length is taken as abscissa.

B. Ideal Indicator Diagram :

1. The graph between the pressure head and stroke length of the piston for one complete revolution of the crank under ideal conditions is known as ideal indicator diagram.
2. As shown in Fig. 4.10.1, different notations are taken as :
   H_{atm} = Atmospheric pressure head = 10.3 m, L = Length of stroke,
   h_s = Suction head, and h_d = Delivery head.
3. During suction stroke, the pressure head in the cylinder is constant and equal to suction head (h_s) which is below the atmospheric pressure head (H_{atm}) by a height of h_s.
4. This pressure head during suction stroke is represented by a horizontal line AB which is below the line EF by the height h_s (suction head).
5. During the delivery stroke, pressure head in cylinder is constant and equal to delivery head (h_d) this is represented by line CD. This line CD is above the line EF (atmospheric pressure) by a height of h_d.

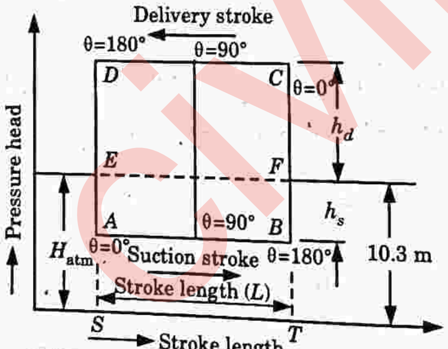


Fig. 4.10.1. Ideal indicator diagram.

Que 4.11. **What do you understand by separation of reciprocating pump ?**

Answer

1. If the pressure in the cylinder is below the vapour pressure, dissolved gasses will be liberated from the liquid and cavitations will takes place. The continuous flow of liquid will not exit which means separation of liquid takes place.
2. The pressure at which separation takes place is called separation pressure and head corresponding to the separation pressure is called separation pressure head.
3. This phenomenon occurs in suction and delivery pipes of reciprocating pumps.
4. The speed at which separation may take place can be increased by fitting an air vessel.

Que 4.12. **Show that the maximum inertia head in a reciprocating pump without air vessel is given by,**

$$H_a = \frac{l}{g} \times \frac{A}{a} \omega_o^2 r$$

AKTU 2016-17, Marks 10

Answer

1. Let us consider the displacement of the piston, after a time t from its inner dead centre (IDC) be x (as shown in Fig. 4.12.1). Then one can write, $x = r - r\cos\theta$
   Where, r is the radius of the crank and θ is the angular displacement of the crank during the time interval t.

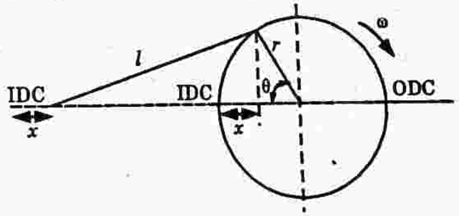


Fig. 4.12.1.

2. Let ω be the angular velocity of the crank. Then we have, $\theta = \omega t$
   Therefore, one can write, $x = r - r\cos\omega t$
3. The velocity of piston is obtained by differentiating the above equation as :
   $$v' = \frac{dx}{dt} = \omega r \sin\omega t \quad ...(4.12.1)$$
4. From continuity equation, we have
   Area of cylinder × Velocity of liquid in cylinder
   = Area of pipe × Velocity of liquid in pipe
   or $AV = av$ [∵ Velocity of liquid in cylinder = Velocity of piston]
   Where A and a are the cross-sectional area of the cylinder and the suction or delivery pipe, respectively and v is the velocity of flow in the pipeline.

or $$v = \frac{A}{a} v'$$

5. Substituting the value $v' = \omega r \sin \omega t$ from eq. (4.12.1), we have
$$v = \frac{A}{a} \omega r \sin \omega t \quad ...(4.12.2)$$
Eq. (4.12.2) provides an expression for the velocity of liquid in the pipeline.
6. The acceleration of liquid in the pipeline is obtained by differentiating eq. (4.12.2) as :
$$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{A}{a}\omega r \sin \omega t\right) = \frac{A}{a}\omega^2 r \cos \omega t \quad ...(4.12.3)$$
7. Mass of liquid flowing in the pipe is given by,
$$m = \text{Density} \times \text{Area of pipe} \times \text{Length of pipe} = \rho a l$$
Where, ρ is the density of liquid and l is the length of the pipe.
8. The force required to accelerate the liquid in the pipe is given by,
$$F = \text{Mass of liquid in pipe} \times \text{Acceleration of liquid in pipe}$$
$$F = \rho a l \times \frac{A}{a}\omega^2 r \cos \omega t$$
9. The intensity of pressure due to the acceleration of liquid in pipe,
$$p_a = \frac{\text{Force required to accelerate liquid}}{\text{Area of pipe}}$$
or $$p_a = \frac{\rho a l \times \frac{A}{a}\omega^2 r \cos \omega t}{a} = \rho l \times \frac{A}{a}\omega^2 r \cos \omega t$$
10. The pressure head (also known as acceleration head) due to the acceleration is given by,
$$H_a = \frac{\text{Intensity of pressure due to the acceleration}}{\text{Specific weight of liquid}}$$
$$= \frac{\rho l \times \frac{A}{a}\omega^2 r \cos \omega t}{\rho g} = \frac{l}{g} \times \frac{A}{a}\omega^2 r \cos \omega t$$
or $$H_a = \frac{l}{g} \times \frac{A}{a}\omega^2 r \cos \theta \quad ...(4.12.4)$$
It is evident from Eq. (4.12.4) that at the middle of the stroke ($\theta = 90°$), the acceleration head is zero and at the beginning ($\theta = 0°$) and end of a stroke ($\theta = 180°$), it becomes maximum and minimum, respectively.
11. The magnitude of the maximum and minimum acceleration head are given by,
$$H_{a\,max} \text{ or } H_{a\,min} = \pm \frac{l}{g} \times \frac{A}{a}\omega^2 r$$

PART-3

Centrifugal Pumps, Operations, Velocity Triangles.

CONCEPT OUTLINE

Rotodynamic Pump : A rotodynamic pump is a kinetic machine in which energy is continuously imparted to the pumped fluid by means of a rotating impeller, propeller or rotor.

Centrifugal Pump : If the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, then the hydraulic machine is called centrifugal pump.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.13. **What is rotodynamic pump ? Explain the classification of rotodynamic pump.**

OR

There are three main categories of dynamic pumps. List and define them.

AKTU 2015-16, Marks 06

Answer

A. **Rotodynamic Pump :** A rotodynamic pump is a kinetic machine in which energy is continuously imparted to the pumped fluid by means of a rotating impeller, propeller or rotor.

B. **Classification :** Dynamic pump can classified as :
1. Radial flow pump. 2. Axial flow pump. 3. Mixed flow pump.

1. **Radial Flow Pump or Centrifugal Pump :**
i. If the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, then the hydraulic machine is called centrifugal pump.
ii. Centrifugal pump is an outward flow machine. It acts as a reverse of an inward radial flow reaction turbine.
iii. Centrifugal pump works on the principle of forced vortex flow *i.e.*, when mass of fluid is rotated by external torque, then pressure head may be raised by $\frac{w^2 x^2}{2g}$.

2. **Axial Flow Type Pump :**
i. The axial flow or propeller pump is the converse of axial flow turbine and is very similar to it in appearance.
ii. The impeller rotates within a cylindrical casing with fine clearance between the blade tips and the casing walls.

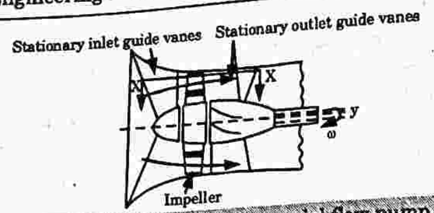


Fig. 4.13.1. A propeller of an axial flow pump.

iii. Fluid particles, in course of their flow through the pump, do not change their radial locations.

iv. The inlet guide vanes are provided to properly direct the fluid to the rotor.

v. The outlet guide vanes are provided to eliminate the whirling component of velocity at discharge.

vi. The usual number of impeller blades lies between 2 and 8, with a hub diameter to impeller diameter ratio of 0.3 to 0.6.

3. Mixed Flow Type Pump :

i. A type of pump that mixes features of radial flow and axial flow pump is called mixed flow pump.

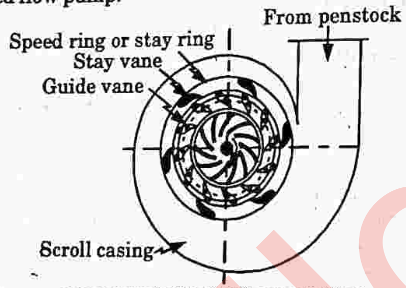


Fig. 4.13.2. Mixed flow pump.

ii. The impeller sits within the pipe and turns, but the turning mechanism is essentially diagonal, using centrifugal force to move the water along while accelerating it further with the push from the axial direction of the impeller. This creates enough force to generate high rates of flow.

iii. The specific speeds (N_s) lie between 35 and 80 rpm for low-speed mixed flow pumps, and between 80 and 160 rpm for higher-speed mixed flow pumps (in special cases even higher).

iv. Mixed flow pump cover the transition range between radial flow pumps and axial flow pumps (e.g., propeller pumps).

v. The impellers of mixed flow pumps with a low specific speed are combined with an annular or volute casing, those of mixed flow pumps with a higher specific speed are combined with a diffuser and a tubular casing.

Que 4.14. **Define centrifugal pump. Explain the classification of centrifugal pump.**

Answer

A. Centrifugal Pump : Refer Q. 4.13, Page 4–20A, Unit-4.

B. Types of Centrifugal Pump : These can be classified as follows :

1. According to the Type of Casing Provided :

i. **Volute Pump :** In a volute pump the impeller is surrounded by a spiral shaped casing known as volute chamber.

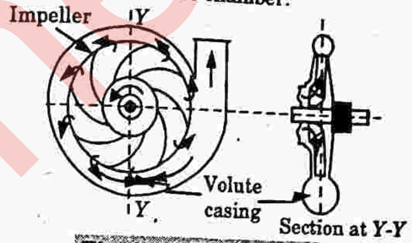


Fig. 4.14.1. Volute pump.

ii. **Diffuser or Turbine Pump :** The impeller is surrounded by a series of guide vanes mounted on a ring called diffuser ring.

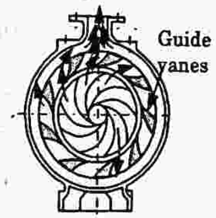


Fig. 4.14.2. Diffuser (or turbine) pump.

2. According to Number of Impeller per Shaft :

i. **Single Stage Centrifugal Pump :** It has only one impeller mounted on the shaft.

ii. **Multi-Stage Centrifugal Pump :** It has two or more impeller connected in series which are mounted on the same shaft and are enclosed in the same casing.

3. According to Relative Direction of Flow of Impeller :

i. **Radial Flow Pump :** In these pump liquid flows through the impeller in the radial direction only. Ordinary centrifugal pumps are generally radial flow type.

ii. **Axial Flow Pump :** It these pump liquid flows through the impeller in the axial direction only. These are designed to deliver very large quantity of liquid at relatively low head.

iii. **Mixed Flow Pump :** In these pump the liquid flow through the impeller axially as well as radially. It is combination of radial flow and axial flow.

4. According to the Number of Entrances to the Impeller :

i. **Single Suction Pump :** In it, liquid is admitted from a suction pipe on one side of the impeller.

ii. **Double Suction Pump :** In it, liquid enters from both side of the impeller thus the axial thrust on the impeller is neutralized and suitable for pumping large quantity of liquids.

5. **According to Disposition of the Shaft :**

i. **Horizontal Disposition of the Shaft :** Generally these pumps are provided with horizontal shaft.

ii. **Vertical Disposition of the Shaft :** For deep wells and mines pumps with vertical shafts are more suitable because the pump with vertically disposed shaft occupy less space.

6. **According to Working Head :**

i. **Low Head Pump :** This type of pump is capable of working against a total head upto 15 m.

ii. **Medium Head Pump :** This type of pump is capable of working against a total head more than 15 m but upto 40 m.

iii. **High Head Pump :** This type of pump is capable of working against a total head above 40 m. Generally high head pumps are multistage pumps.

Que 4.15. **Explain with a neat sketch, the construction details and working principles of a centrifugal pump.**

AKTU 2016-17, Marks 10

Answer

A. **Construction :**

A centrifugal pump consists of the following main components :

1. **Impeller :** An impeller is a wheel (or rotor) with a series of backward curved vanes (or blades). It is mounted on a shaft which is usually coupled to an electric motor.

The impellers are of following three types :

i. **Shrouded or Closed Impeller :** In this type of impeller vanes are provided with metal cover plates or shrouds on both the sides. It is employed when the liquid to be pumped is pure and relatively free from debris.

ii. **Semi-Open Impeller :** A semi-open impeller is one in which vanes have only the base plate and no crown plate. This impeller can be used even if the liquids contain some debris.

iii. **Open Impeller :** In this type the vanes have neither the crown plate nor the base plate *i.e.*, the vanes are open on both sides. Such impellers are employed for pumping liquids which contain suspended solid matter.

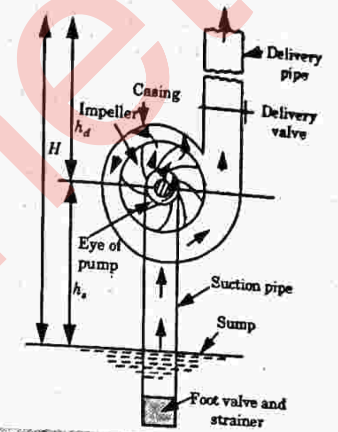


Fig. 4.15.1. Volute types centrifugal pump-component parts.

2. **Casing :** The casing is an airtight chamber surrounding the pump impeller. The essential purposes of the casing are :

i. To guide water to and from the impeller, and

ii. To partially convert the kinetic energy into pressure energy.

The following three types of casing are commonly empolyed :

a. **Volute Casing :** In this type of casing the area of flow gradually increases from the impeller outlet to the delivery pipe so as to reduce the velocity of flow.

b. **Vortex Casing :** If a circular chamber is provided between the impeller and the volute chamber, the casing is known as vortex casing. The circular chamber is known as vortex or whirlpool chamber and such a pump is known as volute pump with vortex chamber.

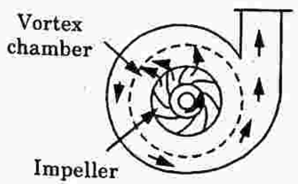


Fig. 4.15.2. Vortex casing.

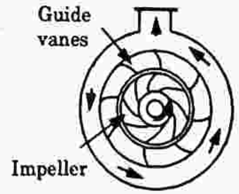


Fig. 4.15.3. Casing with guide blades.

c. **Casing with Guide Blades :** In this type of casing impeller is surrounded by a series or guide blades (or vanes) mounted on a ring which is known as a diffuser.

3. **Suction Pipe :**

i. The pipe which connects the centre/eye of the impeller to sump from which liquid is to be lifted is known as suction pipe.

ii. To prevent the entry of solid particles, debris etc. into the pump the suction pipe is provided with a strainer at its lower end.

4. **Delivery Pipe :**

i. The pipe which is connected at its lower end to the outlet of the pump and it delivers the liquid to the required height is known as delivery pipe.

ii. A regulating valve is provided on the delivery pipe to regulate the supply of water.

B. **Working Principle :**

1. A centrifugal pump works on the principle that when a certain mass of fluid is rotated by an external source, it is thrown away from the central axis of rotation and a centrifugal head is impressed which enables it to rise to a higher level.
2. The working/operation of a centrifugal pump is explained step-wise as follows :

i. The delivery valve is closed and the pump is primed that is, suction pipe, casing and portion of the delivery pipe upto the delivery valve are completely filled with the liquid (to be pumped) so that no air pocket is left.

ii. Keeping the delivery valve still closed the electric motor is started to rotate the impeller. The rotation of the impeller causes strong suction or vacuum just at the eye of the casing.

iii. The speed of the impeller is gradually increased till the impeller rotates at its normal speed and develops normal energy required for pumping the liquid.

iv. After the impeller attains the normal speed, the delivery valve is opened when the liquid is continuously sucked (from sump well) up the suction pipe, it passes through the eye of casing and enters the impellers at its centre or it enters the impeller vanes at their inlet tips.

v. This liquid is impelled out by the rotating vanes and it comes out at the outlet tips of the vanes into the casing. Due to impeller action the pressure head as well as velocity heads of the liquid are increased.

vi. From casing, the liquid passes into pipe and is lifted to the required height (and discharged from the outlet or upper end of the delivery pipe).

vii. So long as motion is given to the impeller and there is supply of liquid to be lifted the process of liquid to the required height remains continuous.

viii. When pump is to be stopped the delivery valve should be first closed, otherwise there may be some backflow from the reservoir.

Que 4.16. **What are the advantages of centrifugal pump over displacement pump ?**

Answer

1. Its discharging capacity is much greater than that of a reciprocating pump.
2. It is compact and had smaller size and weight for the same capacity and energy transfer.
3. Its performance characteristics are superior.

4. It can be employed for lifting highly viscous liquid such as paper pulp, muddy and sewage water, oil, sugar molasses etc.
5. It can be operated at very high speeds without any danger of separation and cavitation.
6. The torque on the power source is uniform, the output from the pump is also uniform.

Que 4.17. **Obtain an expression for the workdone by an impeller of a centrifugal pump on water per second per unit weight of water.**

Answer

1. In case of centrifugal pump, work is done by the impeller on the water. The expression for the workdone by the impeller on the water is obtained by drawing velocity triangles at inlet and outlet of the impeller.
2. Let

N = Speed of impeller.

D_1 and D_2 = Diameter of impeller at inlet and outlet.

u_1 and u_2 = Tangential velocity of impeller at inlet and outlet

$$= \frac{\pi D_1 N}{60} \text{ and } \frac{\pi D_2 N}{60}$$

v_1 and v_2 = Absolute velocity at inlet and outlet.

v_{r1} and v_{r2} = Relative velocity at inlet and outlet.

v_{w1} and v_{w2} = Whirl velocity at inlet and outlet.

v_{f1} and v_{f2} = Flow velocity at inlet and outlet.

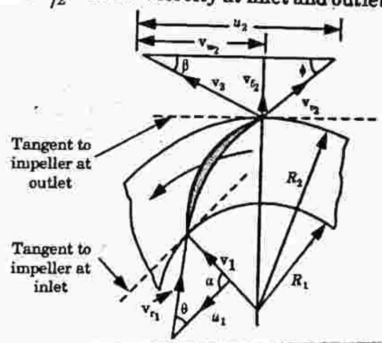


Fig. 4.17.1. Velocity triangles at inlet and outlet.

3. A centrifugal pump is reverse of a radially inward flow reaction turbine.
4. So workdone by the impeller on the water per second per unit weight of water striking per second

$$= - \text{[workdone in case of turbine]}$$

$$= -\left[\frac{1}{g}(v_{w_1} u_1 - v_{w_2} u_2)\right] = \frac{1}{g}\left[(v_{w_2} u_2 - v_{w_1} u_1)\right]$$

5. If water enters radially at inlet, then $v_{w1} = 0 = \dfrac{v_{w_2} u_2}{g}$

Que 4.18. **What is priming ? Why it is necessary ?**

Answer

1. Priming of a centrifugal pump is an operation in which suction pipe, casing of the pump and a portion of delivery pipe is completely filled up with water by an outside source before starting the pump. By doing so, air is removed from these parts.
2. The workdone by impeller per unit weight of liquid per second is known as head generated by the pump. This means that when pump is running in air, the head generated is in terms of meter of air.
3. If pump is primed with water, then head will generate in term of meter of water. But as density of air is low, so head generated by pump is also low even negligible and hence water may not be sucked by the pump.
4. To avoid that difficulty priming of centrifugal pump is necessary.

Que 4.19. **At what height from water surface a centrifugal pump may be installed in the following case to avoid cavitation : atmospheric pressure 101 kPa; vapour pressure 2.34 kPa; inlet and other losses in suction pipe 1.55 m; effective head of pump 52.5 m; and cavitation parameter σ = 0.118.** **AKTU 2016-17, Marks 10**

Answer

Given : Atmospheric pressure, P_a = 101 kPa
Vapour pressure, P_v = 2.34 kPa, Inlet and other losses, h_{fs} = 1.55 m
Effective head (Manometric head), H_{mano} = 52.5 m
Cavitation parameter, σ = 0.118
To Find : Height of water to avoid cavitation.

1. Atmospheric head, $H_a = \dfrac{101 \times 10^3}{9810} = 10.296$ m
2. Vapour head, $H_v = \dfrac{2.34 \times 10^3}{9810} = 0.239$ m
3. Cavitation parameter is given by,

$$\sigma = \frac{H_a - H_s - H_v}{H_{mano}} = \frac{H_a - (h_s + h_{fs}) - H_v}{H_{mano}}$$

$$0.118 = \frac{10.296 - (h_s + 1.55) - 0.239}{52.5} \Rightarrow h_s = 2.312 \text{ m}$$

4. Installation height above water surface, h_s = 2.312 m

PART-4

Performance Curves, Cavitations, Multistaging, Selection of Pumps.

CONCEPT OUTLINE

Head of Pump : The head of pump can be expressed as :
1. Suction head. 2. Delivery head.
3. Static head. 4. Manometric head.

Efficiencies of Pump : The efficiency of pump can be classified as :
1. Manometric efficiency (η_{man}).
2. Mechanical efficiency (η_m).
3. Overall efficiency (η_o).

Cavitation in Pump : If the pressure at the suction side of the pump drops below the vapour pressure of the liquid then the cavitation may occur. The cavitation in a pump can be noted by a sudden drop in efficiency and head.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.20. **Define the terms :**
1. **Suction head.** 2. **Delivery head.**
3. **Static head.** 4. **Manometric head.**

Answer

1. **Suction Head :** It is the vertical height of the centre line of the centrifugal pump above the water surface in sump from which water is to be lifted. It is denoted by h_s.
2. **Delivery Head :** It is the vertical height between the centre line of the pump and the water surface in the tank to which water is delivered. It is denoted by h_d.
3. **Static Head :** It is total vertical height through which water has to be lifted. It is denoted by H_s.

$$H_s = h_s + h_d$$

4. **Manometric Head :** It is defined as the head against which a centrifugal pump has to work. It is denoted by H_m. Manometric head can be calculated by three ways.

i. H_m = Total head at delivery point – Total head at suction point

$$= \left(Z_d + \frac{p_d}{\rho g} + \frac{v_d^2}{2g}\right) - \left(Z_s + \frac{p_s}{\rho g} + \frac{v_s^2}{2g}\right)$$

where, $p_d/\rho g$ and $p_s/\rho g$ = Pressure heads at delivery and suction points respectively and equal to h_d and h_s.

$v_d^2/2g$ and $v_s^2/2g$ = Velocity heads at delivery and suction points.

Z_d and Z_s = Vertical height of delivery and suction point from datum line.

ii. $H_m = (h_s + h_d) + (h_{fs} + h_{fd}) + v_d^2/2g$

h_{fs} and h_{fd} = Frictional losses in suction and delivery pipe.

v_d = Velocity of water in delivery pipe.

iii. By using Bernoulli's equation :

Workdone/sec per unit weight of water = H_m + losses

$$H_m = \frac{v_{w_2} u_2}{g} - \text{losses}$$

If losses are negligible, then $H_m = \dfrac{v_{w_2} u_2}{g}$

Que 4.21. What is NPSH of centrifugal pump ? How it is related to cavitation in pump ?

AKTU 2017-18, Marks 07

Answer

1. **Net Positive Suction Head (NPSH) :** It is defined as the available total suction head at the pump inlet above the head corresponding to the vapour pressure.
2. NPSH = Absolute pressure head at inlet – Vapour pressure head + Velocity head

$$\frac{P_1}{\rho g} - \frac{P_v}{\rho g} + \frac{v_s^2}{2g}$$

3. $\dfrac{P_1}{\rho g}$ is given by, $\dfrac{P_1}{\rho g} = \dfrac{P_a}{\rho g} - \left(\dfrac{v_s^2}{2g} + h_s + h_{fs}\right)$

4. So,

$$\text{NPSH} = \frac{P_a}{\rho g} - \frac{P_v}{\rho g} - h_s - h_{fs}$$

$$\text{NPSH} = H_a - H_v - h_s - h_{fs} \quad \text{...(4.21.1)}$$

5. Thoma's cavitation factor is given by,

$$\sigma = \frac{H_a - H_v - H_s - h_{Ls}}{H} = \frac{H_a - H_v - h_s - h_{fs}}{H_m}$$

$(\because H_s = h_s,\ h_{Ls} = h_{fs}$ and $H = H_m$ for pump$)$

6. From eq. (4.21.1), we get

$$\sigma = \frac{NPSH}{H_m}$$

7. In order to have cavitation free operation of centrifugal pump, the available NPSH should be greater than the required NPSH.

Que 4.22. What do you mean by manometric efficiency, mechanical efficiency and overall efficiency of centrifugal pump ?

Answer

A. **Manometric Efficiency :** The manometric efficiency is defined as the ratio of the manometric head developed by the pump to the head imparted by the impeller to the liquid.

$$\eta_{mano} = \frac{\text{Manometric head}}{\text{Head imparted by impeller to liquid}}$$

$$= \frac{H_m}{(v_{w_2} u_2 / g)} = \frac{gH_m}{(v_{w2} u_2)}$$

or

$$\eta_{mano} = \frac{H_m}{H_m + \text{Losses in the pump}}$$

B. **Mechanical Efficiency :** The mechanical efficiency is defined as the ratio of the power actually delivered by the impeller to the power supplied to the shaft by the prime mover or motor.

$$\eta_{mech} = \frac{\text{Power delivered at impeller}}{\text{Power supplied to the shaft}}$$

Power delivered at impeller (in kW)

$$= \frac{\text{Workdone by impeller per second}}{1000}$$

$$= \frac{W}{g} \times \frac{v_{w_2} u_2}{1000}$$

$$\eta_{mech} = \frac{\frac{W}{g}\left(\frac{v_{w_2} u_2}{1000}\right)}{\text{SP(shaft power)}}$$

C. **Overall Efficiency :** The overall efficiency of the pump is defined as the ratio of the power output from the pump to the power input from the prime mover driving the pump.

$$\eta_o = \frac{\text{Power output}}{\text{Power input}}$$

$$\text{Power output} = \frac{\text{Weight of water lifted} \times H_m}{1000}$$

Power input = Shaft power

$$\therefore \quad \eta_o = \frac{WH_m / 1000}{\text{SP}} \quad \text{or } \eta_o = \eta_{mano} \times \eta_{mech}$$

Que 4.23. Define cavitation. What are the effects of cavitation ? Give the necessary precautions against cavitation.

OR

Define cavitation. And what precautions taken against cavitation.

AKTU 2017-18, Marks 07

Answer

A. Cavitation :

1. Cavitation is defined as the phenomenon of formation of vapour bubbles of a flowing liquid.
2. If in any flow system, the pressure at any point in the liquid approaches the vapour pressure, vaporization of liquid starts, resulting in the formation of the vapour.
3. The bubbles of vapour thus formed are carried by flowing liquid into a region of high pressure where they collapse, giving rise to high impact pressure.
4. The pressure developed by collapsing bubbles is so high that the material is subjected to pitting action. Thus cavities are formed on the metallic surface.

B. Effects of Cavitation are the Effect of Cavitation :

1. The metallic surfaces are damaged and cavities are formed on the surfaces.
2. Due to sudden collapsing of vapour bubbles, considerable noise and vibrations are produced.
3. The efficiency of pump decreases due to cavitation.

C. Precautions : Following precautions can be considered :

1. Pressure of flowing liquid in any part of the pump should not be allowed to fall below its vapour pressure.
2. Cavitation resistant material should be used.

Que 4.24. **Define specific speed of a centrifugal pump. Derive an expression for the same.**

Answer

A. Specific Speed : The specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump which would deliver one cubic meter of liquid per second against a head of one meter. It is denoted by N_s.

B. Expression :

1. Discharge Q for a centrifugal pump is given as :

$$Q = \text{Area} \times \text{Velocity of flow} = \pi D \times B \times v_f \qquad ...(4.24.1)$$

where, D = Diameter of impeller and B = Width of impeller.

2. We know that, $B \propto D$

From eq. (4.24.1), $Q \propto D^2 \times v_f$ ...(4.24.2)

3. Tangential velocity is given as, $u = \dfrac{\pi DN}{60} \Rightarrow u \propto DN$ ...(4.24.3)

4. Tangential velocity (u) and velocity of flow (v_f) are related to the manometric head (H_m) as

$$u \propto v_f \propto \sqrt{H_m} \qquad ...(4.24.4)$$

5. Put the value of u in eq. (4.24.3), we get

$$\sqrt{H_m} \propto DN \Rightarrow D \propto \frac{\sqrt{H_m}}{N}$$

6. Put the value of D in eq. (4.24.2), we get

$$Q \propto \frac{H_m}{N^2} \times v_f \propto \frac{H_m}{N^2} \times \sqrt{H_m} \qquad [v_f \propto \sqrt{H_m}\,]$$

$$Q \propto \frac{H_m^{3/2}}{N^2} \Rightarrow Q = K\frac{H_m^{3/2}}{N^2} \qquad ...(4.24.5)$$

K = Constant of proportionality.

7. If $H_m = 1$ m, $Q = 1$ m³/sec, so $N = N_s$

Substitute these values in eq. (4.24.5), we get

$$1 = K \times \frac{(1)^{3/2}}{N_s^2} \Rightarrow N_s^2 = K$$

8. Put the value of K in eq. (4.24.5), we get

$$Q = N_s^2 \frac{H_m^{3/2}}{N^2}$$

$$\text{Specific speed,} \quad N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}$$

Que 4.25. **Write a note on characteristics curves for rotodynamic pumps.**

AKTU 2014-15, Marks 05

Answer

Following are the characteristics curves of rotodynamic (centrifugal) pumps :

1. **Main Characteristics Curves :**

i. The main characteristic curves of a centrifugal pump consists of variation of head (manometric head, H_m), power and discharge with respect to speed.

ii. For plotting curves of manometric head versus speed, discharge is kept constant.

iii. For plotting curves of discharge versus speed, manometric head (H_m) is kept constant and for plotting curves of power versus speed, the manometric head and discharge are kept constant.

iv. Fig. 4.25.1 shows main characteristic curves of a pump.

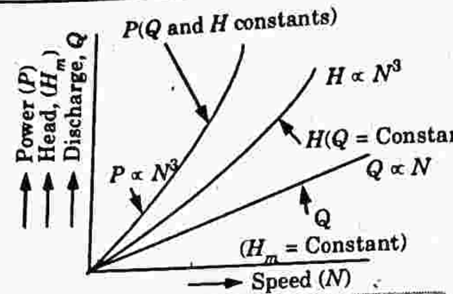


Fig. 4.25.1. Main characteristics curves.

2. **Operating Characteristics Curves :**
   i. If the speed is kept constant, the variation of head, power and efficiency with respect to discharge gives the operating characteristics of pump.
   ii. Different characteristics curves are shown in Fig. 4.25.2.

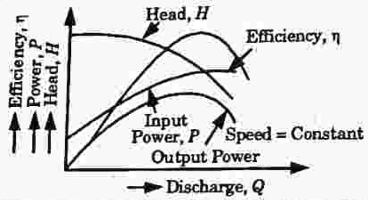


Fig. 4.25.2. Operating characteristics curves of a pump.

3. **Constant Efficiency Curves :**

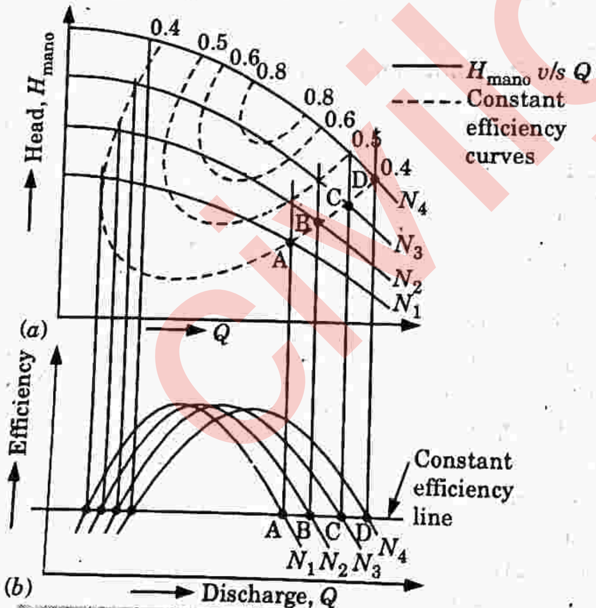


Fig. 4.25.3. Constant efficiency curves of a pump.

i. These curves can be drawn from H_{mano} versus Q and η versus Q curves. Corresponding to a particular value of efficiency two values of discharge are obtained when these data transferred to H_{mano} versus Q, constant efficiency curves.

ii. The curve of best efficiency is obtained on joining the peak points of all these curves.

Que 4.26. **Explain briefly multistage centrifugal pump.**

Answer

Multistage Centrifugal Pump : If centrifugal pump consists of two or more impellers, the pump is called multistage pump. A multistage pump is having two important functions :

1. To produce a high head.
2. To discharge a large quantity of liquid.

1. **Multistage Centrifugal Pump for High Head :**
   i. If a high head is to be developed, the impellers are connected in series or mounted on the same shaft.
   ii. Water enters at inlet of first impeller and discharged with increased pressure at outlet of first impeller.
   iii. This water with increased pressure enters in second impeller and thus increases the head by increasing the pressure.

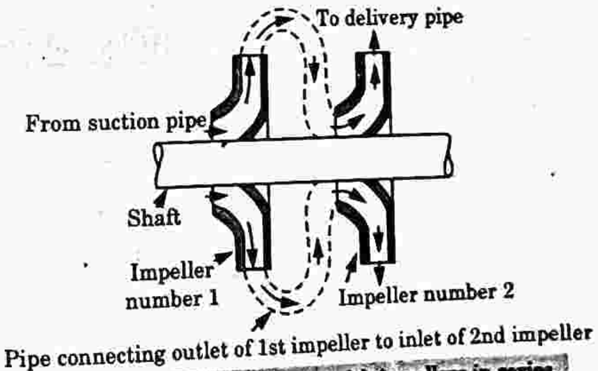


Fig. 4.26.1. Two-stage pump with impellers in series.

2. **Multistage Centrifugal Pump for High Discharge :**
   i. For discharging large quantity of liquid, impellers are connected in parallel.
   ii. Each of the pumps lift the water from common sump and discharge into a common delivery pipe.

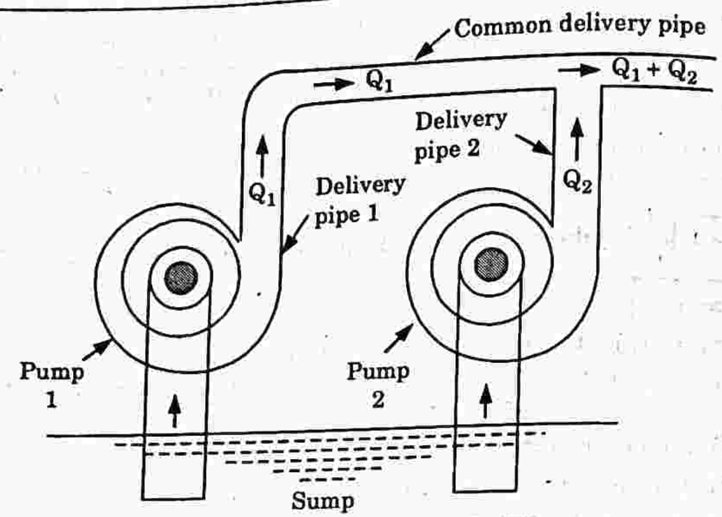


Fig. 4.26.2. Pumps in parallel.

Que 4.27. **A centrifugal pump is running at 1000 rpm. The outlet vane angle of the impeller is 45° and velocity of flow at outlet is 2.5 m/sec. The discharge through the pump is 200 lit/sec when the pump is working against a total head of 20 m. If the manometric efficiency of the pump is 80 %, determine :**

i. Diameter of the impeller (outside diameter).

ii. Width of the impeller at outlet. **AKTU 2015-16, Marks 09**

Answer

Given : Speed, $N = 1000$ rpm, Outlet vane angle, $\phi = 45°$
Velocity of flow at outlet, $v_{f2} = 2.5$ m/sec,
Discharge, $Q = 200$ litres/sec $= 0.2$ m³/sec, Head, $H_m = 20$ m
Manometric efficiency, $\eta_{man} = 80\ \% = 0.80$
To Find : Diameter of impeller width of impeller

1. From outlet velocity triangle, we have

$$\tan \phi = \frac{v_{f_2}}{u_2 - v_{w_2}} \Rightarrow \tan 45° = \frac{2.5}{u_2 - v_{w_2}} \Rightarrow v_{w2} = (u_2 - 2.5) \quad ...(4.27.1)$$

2. Manometric efficiency is given by,

$$\eta_{man} = \frac{gH_m}{v_{w_2} u_2} \Rightarrow 0.80 = \frac{9.81 \times 20}{v_{w_2} u_2} \Rightarrow v_{w2} u_2 = 245.25 \quad ...(4.27.2)$$

3. Substituting the value of v_{w2} from eq. (4.27.1) in (4.27.2), we get

$$(u_2 - 2.5)u_2 = 245.25$$
$$u_2^2 - 2.5u_2 - 245.25 = 0$$
$$\therefore \quad u_2 = 16.96 \text{ m/sec}$$

4. Diameter of impeller, D_2 can be calculated as,

We know that, $u_2 = \dfrac{\pi D_2 N}{60} \Rightarrow 16.96 = \dfrac{\pi \times D_2 \times 1000}{60}$

$\therefore \quad D_2 = 0.324$ m $= 324$ mm

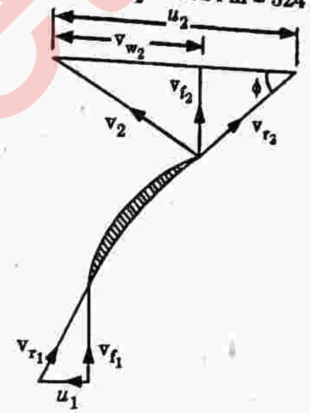


Fig. 4.27.1.

5. Width of impeller at outlet, B_2 can be calculated as,

Discharge, $Q = \pi D_2 B_2 v_{f2} \Rightarrow 0.2 = \pi \times 0.324 \times B_2 \times 2.5$

$B_2 = 0.0786$ m $= 78.6$ mm.

Que 4.28. **A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 rpm works against a total head of 40 m. The velocity of the flow through the impeller is constant and is equal to 2.5 m/sec. The vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 500 mm and width at outlet is 50 mm, determine.**

i. Vane angle at inlet.

ii. Work done by impeller on water per second.

iii. Manometric efficiency. **AKTU 2018-19, Marks 07**

Answer

Given : Speed, $N = 1000$ rpm, Head, $H_m = 40$ m
Velocity of flow, $v_{f1} = v_{f2} = 2.5$ m/sec, Vane angle at outlet, $\phi = 40°$
Outer dia of impeller $D_2 = 500$ mm $= 0.50$ m
Inner dia of impeller, $D_1 = D_2 / 2 = 0.50 / 2 = 0.25$ m
Width at outlet, $B_2 = 50$ mm $= 0.05$ m
To Find : Vane angle at inlet, Work done by impeller, Manometric efficiency.

1. Tangential velocity of impeller at inlet and outlet are

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.09 \text{ m/sec}$$

and
$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.50 \times 1000}{60} = 26.18 \text{ m/sec}$$

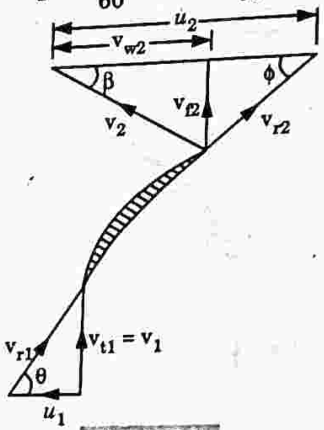


Fig. 4.28.1.

2. Discharge is given by, $Q = \pi D_2 B_2 \times v_{f2}$

$$= \pi \times 0.50 \times 0.05 \times 2.5 = 0.19635 \text{ m}^3\text{/sec}$$

i. Vane angle at inlet (θ).

from inlet velocity triangle, $\tan\theta = v_{f1} / u_1 = 2.5 / 13.09 = 0.191$

$$\theta = \tan^{-1}(0.183) = 10.813°$$

ii. Work done by impeller on water per second is

$$= (W/g) \times v_{w2}\, u_2 = (\rho g Q / g) \times v_{w2} \times u_2$$

$$= 1000 \times 0.19635 \times v_{w2} \times 26.18 \quad ...(4.28.1)$$

But from outlet velocity triangle, we have

$$\tan\phi = \frac{v_{f2}}{u_2 - v_{w2}} \Rightarrow \tan 40° = \frac{2.5}{(26.18 - v_{w2})}$$

$\therefore$ $v_{w2} = 23.2$ m/sec

Substituting this value of v_{w2} in eq. (4.28.1), we get

The work done by impeller = $1000 \times 0.19635 \times 23.2 \times 26.18$

$= 119258.28$ N-m/sec

iii. Manometric efficiency (η_{man}) is given by,

$$\eta_{man} = gH_m / v_{w2}u_2 = 9.81 \times 40 / 23.2 \times 26.18 = 0.646$$

$$\eta_{man} = 64.6\ \%$$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Derive the formula for dynamic force exerted by fluid jet on moving plate for the following cases :

A. When plate is normal to jet.

B. Flat plate inclined to jet.

C. When plate is curved and jet impinges at the center of plate

D. When plate is curved and jet impinges at one end.

Ans. Refer Q. 4.5, Unit-4.

Q. 2. What is reciprocating pump ? Describe the principle and working of a reciprocating pump with a neat sketch.

Ans. Refer Q. 4.7, Unit-4.

Q. 3. There are three main categories of dynamic pumps. List and define them.

Ans. Refer Q. 4.13, Unit-4.

Q. 4. Explain with a neat sketch, the construction details and working principles of a centrifugal pump.

Ans. Refer Q. 4.15, Unit-4.

Q. 5. What is NPSH of centrifugal pump ? How it is related to cavitation in pump ?

Ans. Refer Q. 4.21, Unit-4.

Q. 6. Define cavitation. And what precautions taken against cavitation.

Ans. Refer Q. 4.23, Unit-4.

Q. 7. A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 rpm works against a total head of 40 m. The velocity of the flow through the impeller is constant and is equal is 2.5 m/sec. The vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 500 mm and width at outlet is 50 mm, determine.

i. Vane angle at inlet.

ii. Work done by impeller on water per second.

iii. Manometric efficiency.

Ans. Refer Q. 4.28, Unit-4.

☺☺☺

2. The hydraulic machines, which convert the hydraulic energy into mechanical energy are called turbine while the hydraulic machines which convert the mechanical energy into hydraulic energy are called pump.
3. This mechanical energy is used in running an electric generator which is directly coupled to the shaft of the turbine. Thus the mechanical energy is converted into electrical energy.

B. Classification of Turbine : Hydraulic turbines are classified according to several considerations as follows :

1. According to the Type of Energy at Inlet :

i. Impulse Turbine : In an impulse turbine, all the available energy of water is converted into kinetic energy or velocity head.
Example : Pelton wheel turbine.

ii. Reaction Turbine : In a reaction turbine, at the entrance to the runner, only a part of the available energy of water is converted into kinetic energy and a substantial part remains in the form of pressure energy.
Example : Francis turbine, Kaplan turbine.

2. According to the Direction of Flow through Runner :

i. Tangential Flow Turbine : In this turbine, the water flows along the tangent to the path of rotation of the runner.
Example : Pelton wheel turbine.

ii. Radial Flow Turbine : In this turbine, the water flows along the radial direction.
Example : Francis turbine.

iii. Axial Flow Turbine : In this turbine, water flows through the runner wholly and mainly along the direction parallel to the axis of rotation of the runner.
Example : Kaplan turbine.

iv. Mixed Flow Turbine : In this turbine, water enters the runner at the outer periphery in radial direction and leaves it at the centre in the direction parallel to the axis of rotation of the runner.
Example : Modern Francis turbine.

3. According to Need at Inlet of turbine :

i. High Head Turbine : High head turbines are those which are capable of working under very high head ranging more than 250 m.
Example : Pelton wheel turbine.

ii. Medium Head Turbine : Medium head turbines are those which are capable of working under head ranging from 60 m to 250 m.
Example : Francis turbine.

iii. Low Head Turbine : Low head turbines are those, which are capable of working under head less than 60 m.
Example : Kaplan turbine.

4. According to the Specific Speed of the Turbine :

i. Low Specific Speed Turbine : Low specific speed ranging less than 60.
Example : Pelton wheel turbine.

ii. Medium Specific Speed Turbine : Ranging between 60 to 300.
Example : Francis turbine.

iii. High Specific Speed Turbine : Ranging between 300 to 1000.
Example : Kaplan turbine.

Que 5.3. **Describe the Pelton wheel turbine and explain its components also.**

Answer

A. Pelton Wheel or Pelton Turbine :
1. It is a tangential flow impulse turbine. The water strikes the bucket along the tangent of the runner.
2. The energy available at the inlet of the turbine is only kinetic energy. The pressure at the inlet and outlet of the turbine is atmosphere.
3. The turbine is used for high heads and low discharge.
4. Fig. 5.3.2 shows the layout of a hydro-electric power plant in which the turbine is Pelton wheel.

B. Working :
1. The water from the reservoir flows through the penstock at the outlet of which a nozzle is fitted.
2. The nozzle increases the kinetic energy of the water flowing through the penstock.
3. At the outlet of the nozzle, the water comes out in the form of a jet and strikes the buckets (vanes) of the runner.

C. Components : Main parts of the Pelton turbine are :

1. Nozzle and Flow Regulating Arrangement :
i. The amount of water striking the bucket of runner is controlled by providing a spear in the nozzle. Spear has the streamlined head which is fixed to end of the rod.
ii. When the spear is pushed forward into the nozzle the amount of water striking the runner is reduced when it move pushed backward, the amount of water striking the runner increases.

2. Runner and Bucket :
i. Runner consists of a circular disc with a number of bucket evenly spaced around its periphery.
ii. The shape of a bucket is of a double hemispherical cup or bowl. Each bucket is divided into two symmetrical parts by a sharp edged ridge known as splitter.

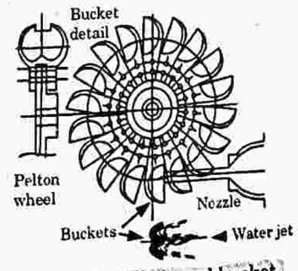


Fig. 5.3.1. Runner and bucket.

iii. The jet of water impinges on the splitter which divides the jet into two equal portions.

iv. The buckets are shaped in such a way that the jet gets deflected through 160° or 170°. The buckets are made of cast iron, cast steel or stainless steel.

3. **Casing :**

i. Fig. 5.3.2 shows the casing of a Pelton wheel turbine. The function of the casing is to prevent the splashing of the water and to discharge water to tail race.

ii. The casing of the Pelton wheel does not perform any hydraulic function.

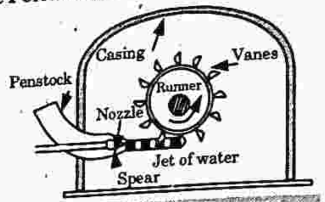


Fig. 5.3.2. Pelton turbine.

4. **Breaking Jet :**

i. When the nozzle is completely closed by the motion of spear in forward direction, the amount of water striking the runner reduces to zero.

ii. But due to inertia, runner goes on revolving. Therefore a jet from back of vane is used to stop the wheel known as breaking jet.

Que 5.4. **Prove that the workdone per second per unit weight of water in Pelton wheel turbine is $\frac{1}{g}[v_{w1} + v_{w2}]\,u$.**

Answer

The inlet velocity triangle is drawn at the splitter and outlet velocity triangle is drawn at outer edge of bucket as shown in Fig. 5.4.1.

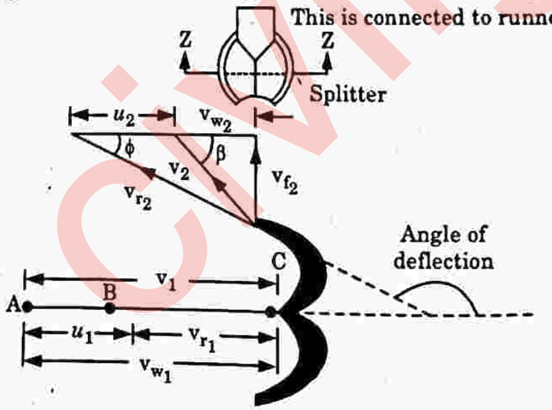


Fig. 5.4.1. Shape of bucket.

Workdone :

Let
H = Net head on Pelton wheel = $H_g - h_f$

H_g = Gross head, $h_f = \frac{4fLv^2}{2gD'}$

D' = Diameter of Penstock

v_1 = Velocity of jet at inlet = $\sqrt{2gH}$

$u = u_1 = u_2 = \frac{\pi DN}{60}$

D = Diameter of wheel.

1. Velocity triangle at inlet will be straight line where

$v_{r1} = v_1 - u_1 = v_1 - u$

(v_{r1} = Relative velocity at inlet)

$v_{w1} = v_1$ (v_{w1} = Whirl velocity at inlet)

$\alpha = 0°$, $\theta = 0°$

α = Guide vane angle at inlet.

θ = Runner blade angle at inlet.

2. Velocity triangle at outlet, $v_{r2} = v_{r1}$, $v_{w2} = v_{r2}\cos\phi - u_2$

v_{r2} = Relative velocity at outlet.

v_{w2} = Whirl velocity at outlet.

3. Force exerted by the jet of water in the direction of motion,

$$F_x = \rho a v_1[v_{w1} + v_{w2}] \quad ...(5.4.1)$$

a = Area of jet = $\frac{\pi}{4}d^2$ [d = Diameter of jet]

4. Workdone by the jet on runner per second

$$F_x \times u = \rho a v_1[v_{w1} + v_{w2}] \times u \text{ N-m/sec} \quad ...(5.4.2)$$

5. Power given to the runner by jet = $\frac{\rho a v_1[v_{w1} + v_{w2}] \times u}{1000}$ kW ...(5.4.3)

6. Workdone/sec per unit weight of water striking per second

$$= \frac{\rho a v_1[v_{w1} + v_{w2}] \times u}{\text{Weight of water striking per second}}$$

$$= \frac{\rho a v_1[v_{w1} + v_{w2}] \times u}{\rho a v_1 \times g} = \frac{1}{g}[v_{w1} + v_{w2}] \times u$$

Que 5.5. **Write down the equation for jet and rotor size of Pelton wheel.**

Answer

Following equations are used for Pelton wheel :

1. The velocity of jet at inlet is given by, $v_1 = C_v\sqrt{2gH}$

Where, C_v = Coefficient of velocity = 0.98 or 0.99

H = Net head on turbine.

2. The velocity of wheel is given by, $u = \phi\sqrt{2gH}$

Where, f = Speed ratio. The value of speed ratio varies from 0.43 to 0.48.

3. The angle of deflection of jet through bucket is taken at 165°. If no deflection angle is given.
4. The mean diameter of Pelton wheel is given by, $u = \frac{\pi DN}{60}$ or $D = \frac{60u}{\pi N}$
5. **Jet Ratio :** It is defined as ratio of mean diameter of Pelton wheel to diameter of jet (d). It is denoted by M.

$$M = D/d, \text{ (12 for most cases)}$$

6. Number of buckets on a runner is given by, $Z = 15 + \frac{D}{2d} = 16 + 0.5\,M$
7. **Number of Jet :** It is obtained by dividing the total rate of flow through the turbine by rate of flow of water through a single jet.

Que 5.6. **Define different types of efficiency of hydraulic turbines.**

AKTU 2017-18, Marks 07

Answer

1. **Hydraulic Efficiency (h_h) :**

i. The ratio of power developed by the runner to the power supplied by the water at the inlet of the turbine is termed as hydraulic efficiency.

ii. It is given by,

$$\eta_h = \frac{\text{Power developed by runner}}{\text{Power supplied to the inlet}}$$

$$= \frac{\text{RP (Runner Power)}}{\text{WP (Water Power)}} = \frac{\rho Q[V_{w1}u_1 \pm V_{w2}u_2]}{wQH/1000}$$

[$\because$ Unit weight of water, $w = \rho g$]

$$\eta_h = \frac{\rho Q[V_{w1}u_1 \pm V_{w2}u_2]}{\rho gQH/1000} = \frac{V_{w1}u_1 \pm V_{w2}u_2}{gH/1000}$$

iii. If velocity of whirl at outlet is zero, $V_{w2} = 0$, *i.e.*, discharge at outlet is radial.

$$\eta_h = \frac{V_{w1}u_1}{gH/1000}$$

2. **Mechanical Efficiency :**

i. It is defined as the ratio of power available at the turbine shaft to the power developed by the runner.

ii. It is given as,

$$\eta_m = \frac{\text{Shaft power}}{\text{Power developed by runner}} = \frac{\text{SP}}{\text{RP}}$$

3. **Overall Efficiency :**

i. It is defined as the ratio of power available at the turbine shaft to the power developed by the water at the inlet of the turbine.

ii. It is given as,

$$\eta_o = \frac{\text{Shaft power}}{\text{Water power}} = \frac{\text{SP}}{\text{WP}} = \frac{P}{\rho gQH/1000}$$

Relationship : Overall efficiency, $\eta_o = \frac{\text{SP}}{\text{WP}} = \frac{\text{SP}}{\text{RP}} \times \frac{\text{RP}}{\text{WP}} = \eta_m \times \eta_h$

Overall efficiency varies between 80 to 90 %.

Que 5.7. **Prove that hydraulic efficiency for Pelton wheel turbine is given by $\eta_h = \frac{2(V_1 - u)(1 + \cos\phi)u}{V_1^2}$ and give the condition for maximum hydraulic efficiency of a Pelton wheel turbine equation for maximum efficiency.**

Answer

1. For Pelton wheel turbine, hydraulic efficiency is given by,

$$\eta_h = \frac{\text{Workdone per second}}{\text{KE of jet per second}}$$

2. Workdone per second = $\rho a V_1[V_{w1} + V_{w2}] \times u$
3. KE of jet per second = $\frac{1}{2} mV_1^2 = \frac{1}{2}(\rho a V_1) \times V_1^2$

So,

$$\eta_h = \frac{\rho a V_1[V_{w1} + V_{w2}] \times u}{\frac{1}{2}(\rho a V_1)V_1^2} \quad ...(5.7.1)$$

2. Now,

$$V_{w1} = V_1,\ V_{r1} = V_1 - u_1 = V_1 - u$$

$$V_{r2} = V_1 - u$$

$$V_{w2} = V_{r2}\cos\phi - u_2 \quad (\phi = \text{Guide vane angle at outlet})$$

$$= V_{r2}\cos\phi - u = (V_1 - u)\cos\phi - u$$

3. Substitute the value of V_{w1} and V_{w2} in eq. (5.7.1), we get

$$\eta_h = \frac{2[V_1 + (V_1 - u)\cos\phi - u] \times u}{V_1^2}$$

$$= \frac{2[V_1 - u + (V_1 - u)\cos\phi] \times u}{V_1^2} = \frac{2(V_1 - u)(1 + \cos\phi)u}{V_1^2}$$

4. For maximum efficiency, $\frac{d}{du}(\eta_h) = 0$,

$$\frac{d}{du}\left[\frac{2(V_1 - u)(1 + \cos\phi)u}{V_1^2}\right] = 0;\ \frac{(1 + \cos\phi)}{V_1^2}\frac{d}{du}[2(V_1 - u)u] = 0$$

$$\frac{d}{du}[2V_1u - 2u^2] = 0;\ 2V_1 - 4u = 0$$

$$u = V_1/2$$

5. So, hydraulic efficiency of a Pelton wheel turbine will be maximum when the velocity of wheel is half the velocity of jet of water at inlet.

Que 5.8. **In a Pelton wheel has a mean bucket speed of 10 m/sec. with a jet of water flowing at the rate of 0.7 m³/sec and a head of**

30 m. If the buckets deflects the jet through an angle of 160°. Calculate the power given by water to the runner and hydraulic efficiency of the turbine. Assume the coefficient of velocity 0.98.

AKTU 2015-16, Marks 10

Answer

Given : Speed of bucket, $u = u_1 = u_2 = 10$ m/sec
Discharge, $Q = 0.7$ m³/sec, Head of water, $H = 30$ m
Angle of deflection = 160°, Angle, $\phi = 180° - 160° = 20°$
Coefficient of velocity, $C_v = 0.98$
To Find : Power transfer and hydraulic efficiency.

1. The velocity of jet, $v_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 30} = 23.77$ m/sec
$$v_{r1} = v_1 - u_1 = 23.77 - 10 = 13.77 \text{ m/sec}$$
$$v_{w1} = v_1 = 23.77 \text{ m/sec}$$
2. From outlet velocity triangle,
$$v_{r2} = v_{r1}\cos\phi - u_2 = 13.77 \times \cos 20° - 10.0 = 2.94 \text{ m/sec}$$
3. Workdone by the jet per second on the runner is given by,
$= \rho a v_1 [v_{w1} + v_{w2}] \times u = 1000 \times 0.7 \times [23.77 + 2.94] \times 10$
$= 186970$ N-m/sec $\qquad (\because \; av_1 = Q = 0.7 \text{ m}^3/\text{sec})$

$\therefore$ Power given to turbine $= \dfrac{186970}{1000} = 186.97$ kW

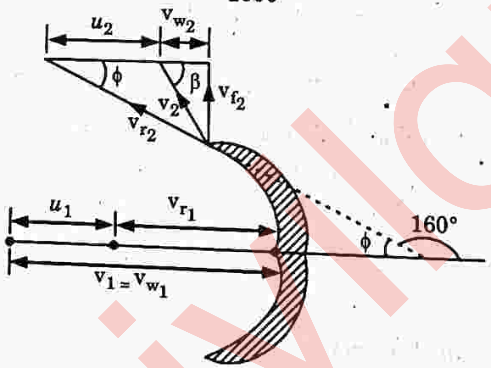


Fig. 5.8.1.

4. The hydraulic efficiency of the turbine is given by,
$$\eta_h = \frac{2[v_{w_1} + v_{w_2}] \times u}{v_1^2} = \frac{2[23.77 + 2.94] \times 10}{23.77 \times 23.77}$$
$= 0.9454$ or 94.54%

Que 5.9. **A Pelton wheel turbine is to be designed for the following specifications : Shaft power = 11,775 kW, Head = 400 m, speed = 750 rpm, Overall efficiency = 86 %, Jet diameter to wheel diameter ratio is not to exceed one sixth (1/6). Find.**

1. **The wheel diameter,** 2. **Diameter of the jet, and**
3. **The number of jets required.**

Assume $K_{v1} = 0.985$ and $K_{u1} = 0.45$

AKTU 2014-15, Marks 10

Answer

Given : Shaft power, SP = 11,775 kW, Head, $H = 400$ m
Speed, $N = 750$ rpm, Overall efficiency, $\eta_o = 0.86$ or 86 %
Ratio of jet diameter to wheel diameter, $d/D = 1/6$
Coefficient of velocity, $C_v = K_{v1} = 0.985$, Speed ratio, $K_{u1} = 0.45$
To Find : Wheel diameter, Diameter of jet, number of jet.

1. **Wheel Diameter :**
i. Velocity of jet, $\quad v_1 = C_v\sqrt{2gH} = 0.985\sqrt{2 \times 9.81 \times 400} = 87.26$ m/sec
ii. Velocity of wheel, $\quad u = u_1 = u_2 = \text{Speed ratio} \times \sqrt{2gH}$
$$u = 0.45 \times \sqrt{2 \times 9.81 \times 400} = 39.87 \text{ m/sec}$$
But, $\quad u = \pi DN/60 \Rightarrow 39.87 = \pi \times D \times 750/60$
$$D = 1.01 \text{ m}$$
2. **Diameter of Jet :**
We know that, $d/D = 1/6 \Rightarrow d = (1/6) \times 1.01 = 0.17$ m
3. Discharge of one jet, q = Area of jet × Velocity of jet
Discharge, $q = (\pi/4)\, d^2 \times v_1 = \pi/4 \times (0.17)^2 \times 87.26 = 1.98$ m³/sec
4. Jet efficiency, $\quad \eta_o = \dfrac{SP}{WP} = \dfrac{11775}{\rho g Q H/1000}$
$$0.86 = \frac{11775 \times 1000}{1000 \times 9.81 \times Q \times 400} \Rightarrow Q = 3.49 \text{ m}^3/\text{sec}$$
5. $\quad$ Number of jets $= \dfrac{\text{Total discharge}}{\text{Discharge of one jet}}$

Number of jets required, $n = 3.49/1.98 = 1.76 \simeq 2$ jets.

Que 5.10. **A pelton wheel is to be designed for the following specification. Shaft power = 11722 kW, Head = 380 mtr, speed = 750 rpm, η_0 = 86 % jet diameter (d) not to exceed one-sixth of wheel diameter. Determine (i) The wheel diameter (ii) Number of jet required (iii) Diameter of jet, take velocity ratio $K_{v1} = 0.985$ and speed ratio $K_{u1} = 0.45$.**

AKTU 2017-18, Marks 07

Answer

Given : Shaft power, $SP = 11722$ kW, Head = 380 m, Speed, $N = 750$ rpm, $\eta_0 = 86$ %, Jet diameter = $d/D = 1/6$, $K_{v1} = 0.985$, $K_{u1} = 0.45$
To Find : Wheel diameter, Diameter of jet, Number of jet.

1. **Wheel Diameter :**

i. Velocity of jet, $v_1 = K_{v1}\sqrt{2gH} = 0.985\sqrt{2\times 9.81\times 380} = 85.05$ m/sec

ii. Velocity of wheel, $u_1 = u_2 =$ Speed ratio $\times \sqrt{2gH}$

$$= 0.45\sqrt{2\times 9.81\times 380} = 38.86 \text{ m/sec}$$

But, $u = \pi DN/60$

$$38.86 = \pi \times D \times \frac{750}{60}$$

$$D = 0.988 \approx 1 \text{ m}$$

2. **Diameter of Jet :**

We know that, $\dfrac{d}{D} = \dfrac{1}{6}$

$$d = \frac{1}{6}\times 1 = 0.17 \text{ m}$$

3. Discharge of one jet, q = Area of jet × Velocity of jet

$$q = \frac{\pi}{4}(d^2)\times v_1 = \frac{\pi}{4}\times(0.17)^2\times 85.05$$

$$q = 1.93 \text{ m}^3/\text{sec}$$

4. Jet efficiency, $\eta_0 = \dfrac{\text{Shaft power}}{\text{Water power}} = \dfrac{11722}{\rho\, gQH/1000}$

$$0.86 = \frac{11722\times 1000}{1000\times 9.81\times 380\times Q}$$

Total discharge, $Q = 3.66$ m³/sec

5. Number of jet $= \dfrac{\text{Total discharge}}{\text{Discharge of one jet}} = \dfrac{3.66}{1.93} = 1.89 \approx 2$

Que 5.11. **Design a Pelton wheel which is required to develop 1500 kW, when working under a head of 160 m at a speed of 420 rpm. The overall efficiency may be taken as 85 % and assume other data required.**

AKTU 2016-17, Marks 10

Answer

Given : Shaft power, $P = 1500$ kW, Head, $H = 160$ m
Speed, $N = 420$ rpm, Overall efficiency, $\eta_0 = 85$ %
Assume coefficient of velocity, $C_v = 0.98$
Velocity of bucket, $u = 0.45$ times velocity of jet
To Find : Design of Pelton wheel.

Design of Pelton wheel means to find :

1. Diameter of jet.
2. Velocity of jet.
3. Width and depth of bucket.
4. Number of buckets on the wheel.

1. **Velocity of Jet :**

$$v_1 = C_v \times \sqrt{2gH} = 0.98\times\sqrt{2\times 9.81\times 160} = 54.90 \text{ m/sec}$$

∴ Bucket velocity, $u = u_1 = u_2 = 0.45\ v_1 = 24.70$ m/sec

But $u = \pi DN/60$

$$24.70 = \pi \times D \times 420/60$$

Diameter of wheel, $D = 1.12$ m

2. **Diameter of the Jet, d :**

Overall efficiency, $\eta_o = \dfrac{\text{Shaft power}}{\text{Water power}}$

$$0.85 = \frac{1500\times 10^3}{\rho g Q H} = \frac{1500\times 10^3}{1000\times 9.81\times Q\times 160}$$

$$Q = 1.124 \text{ m}^3/\text{sec}$$

But discharge, Q = Area of jet × Velocity of jet

$$1.124 = \frac{\pi}{4}\times d^2 \times 54.90 \Rightarrow d = 0.16 \text{ m} = 16 \text{ cm}$$

3. **Size of Buckets :** Width of bucket $= 5\times d = 5\times 16 = 80$ cm
Depth of bucket $= 1.2\times d = 1.2\times 16 = 19.2$ cm

4. **Number of Buckets on the Wheel :**
$N = 15 + D/2d = 15 + 1.12/(2\times 0.16) = 18.5 \approx 19$

PART-2

Reaction Turbines, Francis and Kaplan Type, Heads on Reaction Turbine.

CONCEPT OUTLINE

Reaction Turbine : If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy, the turbine is known as reaction turbine.

Types of Reaction Turbine :
1. Francis turbine
2. Kaplan turbine

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.12. **Explain the term reaction turbines and classify them.**

Answer

1. If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy, the turbine is known as reaction turbine.
2. In reaction turbine only a part of the total energy is converted into the kinetic energy. The rest pressure of the flowing water changes gradually as it passes through the runner.

3. Therefore the runner must be enclosed within a water tight casing.
4. There is a difference of pressure between the guide vanes and the runner called as reaction pressure which is responsible for the motion of runner.
5. Reaction turbine can be classified as :
   i. Radial flow reaction turbine (Francis turbine).
   ii. Axial flow reaction turbine (Kaplan turbine).
6. Radial flow reaction turbine is further classified as :
   i. Inward flow turbine. ii. Outward flow turbine.
7. If water flow from outward to inward, radially, the turbine is known as inward flow turbine otherwise it is known as outward flow turbine.

Que 5.13. Explain the Francis turbines and describe briefly various components of reaction turbines.

Answer

A. Francis Turbine :

1. It is a mixed flow reaction turbine. It is suitable for medium head and medium discharge. Francis first developed an inward radial flow reaction turbine and later it was modified as mixed flow type, which is known as modern Francis turbine.
2. In a Francis turbine fluid enters from penstock to a spiral casing which completely surrounds the runners as shown in (Fig. 5.13.1).

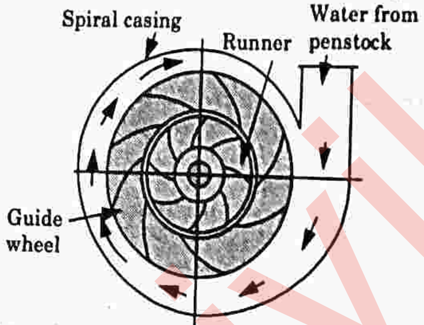


Fig. 5.13.1. Main parts of a radial flow reaction turbine.

B. Components : The main parts of a radial flow reaction turbine are as follows :

1. **Casing :** It is made of spiral in shape and area is reduced gradually so that flow velocity can be kept constant throughout the circumference.
2. **Guide Mechanism :**
   i. It consists of a stationary circular wheel all around the runner of the turbine.
   ii. The stationary guide vanes are permanently fixed to the casing which guides the water to enter into the runner.

3. **Runner :**
   i. Runner is a rotating unit. It is main part of turbine.
   ii. It is a circular wheel on which a series of radial curved vanes are fixed.
   iii. Surface of vane is very smooth to avoid friction losses.
4. **Draft Tube :**
   i. A tube or pipe of gradually increasing area is used for discharging water from exit of turbine to the tail race. This tube of increasing area is called draft tube.
   ii. The primary function of the draft tube is to minimize the velocity of discharge, thus, minimizing the loss of kinetic energy at the outlet.
5. **Penstock :** It is the pipe which brings the water either from surge tank or from reservoir.

Que 5.14. Explain the velocity triangle for reaction turbine.

Answer

A. Velocity Triangle :

1. Let water coming from casing is guided at an angle a from wheel tangent by guide blade. Hence a is called as guide blade angle (Fig. 5.14.1).
   Let v_1 is velocity of water leaving the guide vane and entering into runner blade. This is called absolute velocity (v_1) of water at inlet.
   Now v_1 has two components :
   i. $$v_1 \cos\alpha = v_{w1}$$
   It is the tangential component in the direction of u_1. It is called velocity of whirl at inlet. It is responsible for rotation of wheel.
   ii. $$v_1 \sin\alpha = v_{f1}$$
   It is radial component of v_1 and called flow or radial velocity at inlet.
   Now, $u_1 = r_1 w = \pi d_1 N / 60$
   d_1 = Diameter at inlet
   u_1 is called tangential or peripheral velocity at inlet. It is along the wheel tangent.
   v_{r1} is the relative velocity at inlet and is along the surface of vane.
   θ = Runner blade angle at inlet, and
   α = Guide vane angle at inlet.

B. Conditions of Inlet Velocity Triangle :

1. When $\theta < 90°$, velocity triangle is made as,
   Here, $$\tan\theta = \frac{v_{f_1}}{v_{w_1} - u_1} = \frac{v_{f_1}}{v_{w_1}}$$
2. When $\theta = 90°$, velocity triangle is made as
   In this case, $v_{r1} = v_{f1}; u_1 = v_{w1}; v_{w2} = 0$
   and $\tan\alpha = (v_{r1}/u_1) = (v_{f1}/u_1)$
3. When q > 90°
   Here, $$\tan\alpha = \frac{v_{f_1}}{v_{w_1}}; \tan(180° - \theta) = \frac{v_{f_1}}{u_1 - v_{w_1}}$$

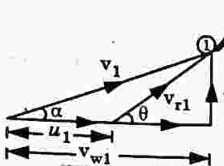


Fig. 5.14.1.

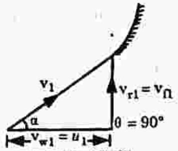


Fig. 5.14.2.

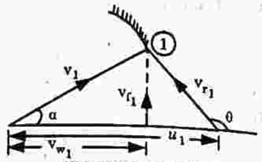


Fig. 5.14.3.

C. Conditions for outlet velocity :

1. **When u_2 and v_{w2} are in Same Direction :**
2. **When $v_{w2} = 0$, $\beta = 90°$**
   In this case, discharge is said to be radial at exit or water leaves the runner vane without whirl ($v_{w2} = 0$).
   In this case, we achieve maximum work done.
3. **When v_{w2} is in Opposite Direction of v_2 :**

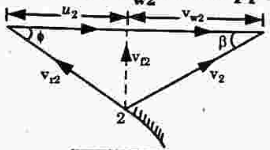


Fig. 5.14.4.

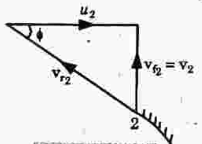


Fig. 5.14.5.

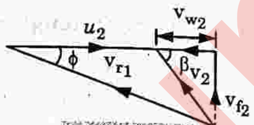


Fig. 5.14.6.

Que 5.15. The specific speed of the high speed Francis turbine is 200 rpm. Total power generated in the hydro-electric power station is 2000 kW and the head of water available is 18 m and the turbines are to run at 250 rpm. Determine numbers of turbines that are required in the power station.

Answer

Given : Specific speed of turbine, $N_S = 200$ rpm,
Total power generated = 2000 kW, Speed of turbine, $N = 250$ rpm
Head of water, $H = 18$ m
To Find : Number of turbine.

1. Specific speed is given by, $N_S{}^2 = \dfrac{N^2 P}{H^{5/2}}$; $P = \dfrac{N_S^2 H^{5/2}}{N^2}$

$$P = (200^2 \times 18^{5/2}) / (250)^2 = 879.75 \text{ kW}$$

2. Number of turbines = 2000 / 879.95 = 2.27 = 3 turbines

Que 5.16. Explain the axial flow reaction (Kaplan) turbine with its working.

Answer

A. Axial Flow Reaction Turbine :

1. If the water flows parallel to the axis of the rotation of the shaft, the turbine is known as axial flow turbine. And if the head at the inlet of the

turbine is the sum of pressure energy and kinetic energy and during the flow of water through runner a part of pressure energy is converted into kinetic energy, the turbine is known as reaction turbine.

2. For the axial flow reaction turbine, shaft of the turbine is vertical. The lower end of the shaft is made larger which is known as 'hub' or 'boss'. The vanes are fixed on the hub and hence act as a runner for axial flow reaction turbine.
3. The following are the important type of axial flow reaction turbines :
   i. Propeller turbines, and
   ii. Kaplan turbine.
4. When the vanes are fixed to the hub and they are not adjustable, the turbine is known as propeller turbine. But if the vanes on the hub are adjustable the turbine is known as a Kaplan turbine. This turbine is suitable where a large quantity of water at low heads is available.

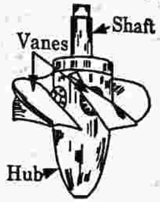


Fig. 5.16.1. Kaplan turbine runner.

5. Fig. 5.16.1 shows the runner of a Kaplan turbine which consists of a hub fixed to the shaft. On the hub, the adjustable vanes are fixed.
6. The following are the important points for propeller or Kaplan turbine :
   i. The peripheral velocity at inlet and outlet are equal

$$\therefore \quad u_1 = u_2 = \frac{\pi D_0 N}{60}$$

   where, D_0 = Outer diameter of runner

   ii. Velocity of flow at inlet and outlet are equal, $v_{f_1} = v_{f_2}$
   iii. Area of flow at inlet = Area of flow at outlet

$$= \frac{\pi}{4}(D_0^2 - D_b^2) \qquad [\because D_b = \text{Diameter of hub}]$$

B. Components : The main parts of Kaplan turbine are :

1. Scroll casing.
2. Guide vanes mechanism.
3. Hub with vanes or runner of the turbine.
4. Draft tube.

C. Working :

1. Fig. 5.16.2 shows all main parts of a Kaplan turbine. The water from penstock enters the scroll casing and then moves to the guide vanes.

2. From the guide vanes, the water turns through 90° and flows axially through the runner.
3. The discharge through the runner is obtained as,

$$Q = \frac{\pi}{4}(D_0^2 - D_b^2) \times V_{f_1}$$

where, D_b = Diameter of hub.

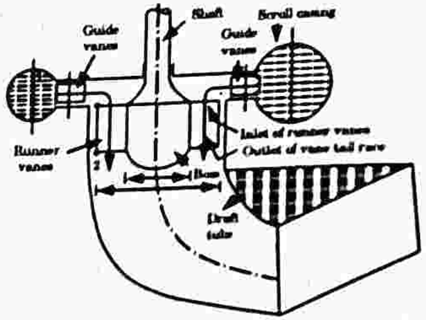


Fig. 5.16.2. Main components of Kalpan turbine.

Que 5.17. Draw neat sketch of various shapes of draft tubes. Also, explain the theory of draft tube. **AKTU 2014-15, Marks 10**

OR

Draw neat sketches of various shapes of draft tubes. **AKTU 2015-16, Marks 05**

OR

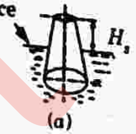
What is a draft tube ? Write neat sketch, list the different types of draft tube. **AKTU 2018-19, Marks 07**

Answer

A. **Draft Tube :** It is a gradually expanding tube which discharges water, passing through the runner to the tail race. The primary function of the draft tube is to minimize the velocity of discharge, thus, minimizing the loss of kinetic energy at the outlet.

B. **Types of Draft Tube :** Following are the important type of draft tubes which are commonly used.

1. **Conical Draft Tube :**



(a)

2. **Simple Elbow Type Draft Tube :**

(b)

3. **Moody Spreading Tube :**

(c)

4. **Draft Tube with Circular Inlet and Rectangular Outlet :**

(d)

Fig. 5.17.1. Types of draft tube.

C. **Draft Tube Theory :**

1. Consider a draft tube as shown in Fig. 5.17.2.

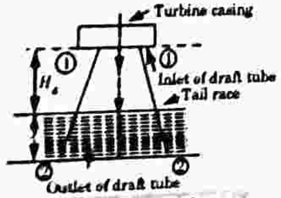


Fig. 5.17.2. Draft tube theory.

Let, H_s = Vertical height of tube above the tail race.

y = Distance of bottom of tube from tail race.

2. Applying Bernoulli's equation at section (1)-(1) and (2)-(2) : (Assuming section (2)-(2) as datum)

$$(P_1/\rho g) + (v_1^2/2g) + (H_s + y) = (P_2/\rho g) + (v_2^2/2g) + 0 + h_f \quad ...(5.17.1)$$

where, h_f = Loss of energy between sections (1)-(1) and (2)-(2)

But, $P_2/\rho g$ = Atmospheric pressure + Head

$$(P_2/\rho g) = (P_{atm}/\rho g) + y$$

3. Substituting this value of $P_2/\rho g$ in eq.(5.18.1), we get,

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + H_s + y = \frac{P_{atm}}{\rho g} + y + \frac{v_2^2}{2g} + h_f$$

or, $$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + H_s = \frac{P_{atm}}{\rho g} + \frac{v_2^2}{2g} + h_f$$

$$\frac{P_1}{\rho g} = \frac{P_{atm}}{\rho g} - H_s - \left(\frac{v_1^2}{2g} - \frac{v_2^2}{2g} - h_f\right) \quad ...(5.17.2)$$

4. From eq. (5.17.2), it is prove that,

$$P_{atm}/\rho g > P_1/\rho g$$

Que 5.18. **Differentiate between impulse and reaction turbine, radial and axial flow turbine.** **AKTU 2014-15, Marks 05**

Answer

A. Difference between Radial Flow/Francis Turbine and Axial Flow/Kaplan Turbine :

S.No.	Aspects	Radial Flow (Francis) Turbine	Axial Flow (Kaplan) Turbine
1.	Type of turbine	Radially inward or mixed flow	Partially axial flow
2.	Vanes	Large, 16 to 24 blades	Small, 3 to 8 blades
3.	Resistance to be overcome	Large, (owing to large number of vanes and greater area of contact with water)	Less (owing to fewer number of vanes and less wetted area)
4.	Disposition of shaft	Horizontal or vertical	Only vertical
5.	Adjustability of runner	Runner vanes are not adjustable.	Runner vanes are adjustable.
6.	Specific speed	60–300 rpm	300-1000 rpm
7.	Type of governor.	Ordinary	Heavy duty
8.	Flow rate	Medium	Large
9.	Head	Medium (60 m to 250 m)	Low (upto 60m)

B. Comparison between Impulse and Reaction Turbines :

S.No.	Aspects	Impulse Turbine	Reaction Turbine
1.	Conversion of fluid energy.	The available fluid energy is converted into kinetic energy by a nozzle.	The energy of the fluid is partly transformed into kinetic energy before it (fluid) enters the runner of the turbine.
2.	Changes in pressure and velocity.	The pressure remains same (atmospheric) throughout the action of water on the runner.	After entering the runner with an excess pressure, water undergoes changes both in velocity and pressure while passing through the runner.
3.	Admittance of water over the wheel.	Water may be allowed to enter a part or whole of the wheel circumference.	Water is admitted over the circumference of the wheel.
4.	Water-tight casing	Required	Not necessary.
5.	Extent to which the water fills the wheel/turbine.	The wheel/turbine does not run full and air has a free access to the buckets.	Water completely fills all the passages between the blades and while flowing between inlet and outlet sections does work on the blades.
6.	Installation of unit.	Always installed above the tail race. No draft tube is used.	Unit may be installed above or below the tail race, use of a draft tube is made.
7.	Relative velocity of water.	Either remaining constant or reduces slightly due to friction.	Due to continuous drop in pressure during flow through the blades, the relative velocity increases.
8.	Flow regulation.	By means of a needle valve fitted into the nozzle. Impossible without loss.	By means of a guide-vane assembly. Always accompanied by loss.

Que 5.19. **An inward flow reaction turbine is required to develop 300 kW at 200 rpm. The active head at the turbine is 18 m. Determine the outside and inside diameters, the inlet and exit angles for the vanes and the exit angle for the guide vanes. Assume the inlet diameter equal to twice the outlet diameter, the hydraulic efficiency as 80 %, the constant radial velocity of flow of 3.6 m³/sec through the runner, the mechanical efficiency as 95 % and the width ratio as 0.1, water leaves the runner radially.**

AKTU 2016-17, Marks 15

Answer

Given : Shaft power = 300 kW, Speed of runner, N = 200 rpm
Active head, H = 18 m, Inlet diameter = 2 × Outlet diameter
Hydraulic efficiency, η_h = 80 %, Discharge, Q = 3.6 m/sec
Mechanical efficiency = 95 %, Width ratio = 0.1
To Find : Inside and outside diameter of vanes, exist angle for vanes and guide vanes.

1. **Main Dimensions of the Turbine :**

i. Shaft power, $P = wQH \times \eta_0$

Overall efficiency, $\eta_0 = \eta_m \times \eta_h = 0.80 \times 0.95 = 0.76$

$\therefore$ Discharge $Q = \dfrac{P}{wH\eta_0} = \dfrac{300}{9.81 \times 18 \times 0.76} = 2.23 \text{ m}^3/\text{sec}$

ii. We know that $Q = \pi D_1 B_1 V_{f1}$

where D_1 and B_1 are the diameter and width of the wheel at the inlet respectively.

$$2.23 = \pi D_1 \times 0.1\, D_1 \times 3.6 \qquad (\because\ B_1/D_1 = 0.1)$$

$$D_1 = 1.40 \text{ m and } B_1 = 0.1 \times 1.40 = 0.14 \text{ m}$$

iii. Diameter of wheel at outlet, $D_2 = 0.5\, D_1 = 0.5 \times 1.40 = 0.70$ m

2. **Angles at Inlet :**

i. Peripheral velocity at inlet, $u_1 = \dfrac{\pi D_1 N}{60} = \dfrac{\pi \times 1.4 \times 200}{60} = 14.66$ m/sec

ii. Hydraulic efficiency, $\eta_h = \dfrac{v_{w1} u_1}{gH} \Rightarrow 0.80 = \dfrac{v_{w1} \times 14.66}{9.81 \times 18}$

$$v_{w1} = 9.63 \text{ m/sec}$$

Since $u_1 > v_{w1}$, the inlet triangle will be as shown in Fig. 5.19.1.

iii. From inlet triangle, $\tan \alpha = \dfrac{v_{f1}}{v_{w1}} = \dfrac{3.6}{9.63}$

$\therefore$ Guide vane angle, $\alpha = 20.49°$

iv. Again, $\tan(180° - \theta) = \dfrac{v_{f1}}{u_1 - v_{w1}} = \dfrac{3.6}{14.66 - 9.63} = 0.715$

$$180° - \theta = 35.60°$$

$\therefore$ Vane inlet angle, $\theta = 144.40°$

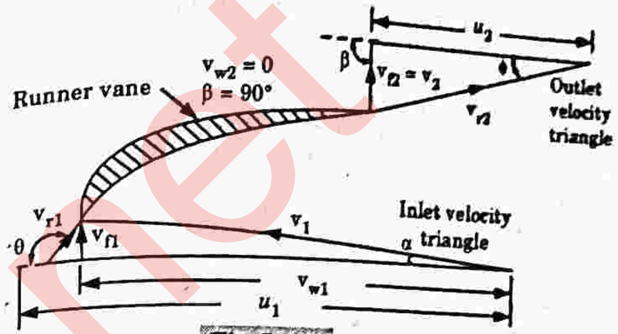


Fig. 5.19.1.

3. **Angles at Outlet :**

i. From outlet triangle, $u_2 = \dfrac{\pi D_2 N}{60} = \dfrac{\pi \times 0.70 \times 200}{60} = 7.33$ m/sec

ii. Discharge is radial, hence, $\beta = 90°$

$$\tan \phi = \frac{v_{f2}}{u_2} = \frac{3.6}{7.33} \qquad [\because v_{f1} = v_{f2}]$$

$\therefore$ Vane angle at outlet, $\phi = 26.15°$

Que 5.20. **Explain the various heads of turbines.**

Answer

1. **Gross Head :** It is difference between water level at the reservoir (head race) and water level at the tail race. It is represented by H_g.
2. **Net or Effective Head :** It is the head available at the inlet of the turbine it and denoted by H.

$$H = H_g - h_f$$

Where, h_f = Head loss due to friction between penstock and water = $\dfrac{4fLv^2}{D \times 2g}$

v = Velocity of flow at penstock.
L = Length of penstock.
D = Diameter of penstock.

PART-3

Unit Quantities, Similarity Laws and Specific Speed, Cavitation, Characteristic Curves.

CONCEPT OUTLINE

Unit Quantities : In order to predict the behaviour of a turbine working under varying conditions of head, speed, output and gate opening, the results are expressed in terms of quantities which may be obtained when the head on the turbine is reduced to unity.

Similitude : Similitude is defined as the similarity between the model and its prototype in every respect, which means that the model and prototype have similar properties or model and prototype are completely similar.

Specific Speed : It is defined as the speed of a turbine which is identical in shape, geometrical dimensions, blade angles, gate opening etc., with the actual turbine but of a size that it will develop unit power when working under unit head.

Cavitation : The formation, growth and collapse of vapour filled cavities or bubbles in a flowing liquid due to local fall in fluid pressure is called cavitation.

Characteristic Curves : Characteristic curves of a hydraulic turbine are the curves, with the help of which the exact behaviour and performance of the turbine under different working conditions, can be known.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.21. **Define the terms unit power, unit speed and unit discharge. Obtain an expression for unit speed.**

Answer

For analyzing the working of turbine under stable conditions, head on turbine is reduced to unity. The conditions of turbine under unit head are such that the efficiency of the turbine remains unaffected. The following three quantities must be studied under unit head :

1. Unit speed 2. Unit discharge 3. Unit power.

1. Unit Speed :

i. It is defined as the speed of a turbine working under a unit head (*i.e.*, under a head of 1 m). It is denoted by N_u.

ii. Let N = Speed of turbine under a head H.

H = Head under which a turbine is working.

u = Tangential velocity.

iii. The tangential velocity, absolute velocity of water and head on the turbine are related as :

$$u \propto v \propto \sqrt{H} \qquad (\because\ v \propto \sqrt{H}) \qquad ...(5.21.1)$$

iv. Tangential velocity also given as :

$$u = \frac{\pi DN}{60}$$

where, D = Diameter of turbine.

v. For a given turbine, diameter (D) is constant.

$$N \propto u$$

So, $$N \propto \sqrt{H} \qquad \text{[From eq. (5.21.1)]}$$

$$N = K_1\sqrt{H} \qquad ...(5.21.2)$$

K_1 = Constant of proportionality.

vi. If head on turbine is unity, speed becomes unit speed.

$$N_u = K_1\sqrt{1}$$

$$K_1 = N_u$$

vii. Put value of K_1 in eq. (5.21.2), we get

$$N = N_u\sqrt{H}$$

Unit speed, $$N_u = \frac{N}{\sqrt{H}}$$

2. Unit Discharge :

i. It is defined as the discharge passing through a turbine, which is working under a unit head. It is represented by symbol Q_u.

ii. Let H = Head of water on the turbine.

Q = Discharge passing through turbine when head is H.

a = Area of flow of water.

iii. Discharge passing through a given turbine under a head H is given by,

$$Q = \text{Area of flow} \times \text{Velocity} \qquad ...(5.21.3)$$

Area of flow for a given turbine is constant and

$$\text{Velocity} \propto \sqrt{H} \qquad ...(5.21.4)$$

iv. From eq. (5.21.3) and eq. (5.21.4)

$$Q \propto \sqrt{H},\ Q = K_2\sqrt{H} \qquad ...(5.21.5)$$

v. For unit head : $H = 1$, $Q = Q_u$

$$Q_u = K_2\sqrt{1} = K_2$$

vi. Put this value of K_2 in eq. (5.21.5), we get

$$Q = Q_u\sqrt{H}$$

Unit discharge, $$Q_u = Q/\sqrt{H}$$

1. Unit Power :

i. It is defined as the power developed by a turbine, working under a unit head. It is represented by symbol P_u.

ii. Let H = Head of water on turbine.

P = Power developed by the turbine under a head H.

Q = Discharge through turbine under a head H.

iii. Overall efficiency is given as :

$$\eta_0 = \frac{\text{Power developed}}{\text{Water power}} = \frac{P}{\dfrac{\rho \times g \times Q \times H}{1000}}$$

$$P = \eta_o \times \frac{\rho g Q H}{1000} \quad \text{...(5.21.6)}$$

$$P \propto Q \times H \propto \sqrt{H} \times H \quad [\because \; Q \propto \sqrt{H}]$$

$$P \propto H^{3/2}, P = K_3 H^{3/2} \quad \text{...(5.21.7)}$$

iv. For unit head : $H = 1$, $P = P_u$

$$P_u = K_3 (1)^{3/2} = K_3$$

v. Put value of K_3 in eq. (5.21.7), we get

$$P = P_u H^{3/2}$$

Unit power, $P_u = \dfrac{P}{H^{3/2}}$

Que 5.22. **Define the similitude. Explain the different type of similitude.**

Answer

A. **Similitude :** It is defined as the similarity between the model and its prototype in energy respect which means that the model and prototype have similar properties or model and prototype are completely similar.

B. **Types of similarities :** The different types of similarities are given as :

1. **Geometric Similarity :**
   i. If the ratios of all corresponding linear dimensions of the model and prototype are equal, there is a geometric similarity exist between the model and prototype.
   ii. Let 'm' is the subscript for model and 'p' is subscript for prototype. For geometric similarity, L = Length, B = Breadth, A = Area, D = Diameter, V = Volume.

So, $$\frac{L_p}{L_m} = \frac{B_p}{B_m} = \frac{D_p}{D_m} = L_r = \text{Scale ratio}$$

   iii. Similarly area ratio and volume ratio are :

$$A_r = \text{Area ratio} = \frac{L_p \times B_p}{L_m \times B_m} = L_r^2$$

and $$V_r = \text{Volume ratio} = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{B_p}{B_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3 = L_r^3$$

2. **Kinematic Similarity :**
   i. It means the similarity of motion between model and prototype.
   ii. If at the corresponding points in the model and in the prototype, the velocity or acceleration ratios are same (both in magnitude and direction), the two flows are said to be kinematically similar.
   iii. Considering two points 1 and 2 in model and prototype.

v = Velocity and a = Acceleration

$$\frac{(v_1)_m}{(v_1)_p} = \frac{(v_2)_m}{(v_2)_p} = v_r = \text{Velocity ratio}$$

iv. Similarly $$\frac{(a_1)_m}{(a_1)_p} = \frac{(a_2)_m}{(a_2)_p} = a_r = \text{Acceleration ratio}$$

Note : The geometric similarity is a pre-requisite for kinematic similarity.

3. **Dynamic Similarity :**
   i. Dynamic similarity means the similarity of force (both in magnitude and in direction) in model and in prototype.
   ii. Let us consider, F_i = Inertia force, F_v = Viscous force and F_g = Gravity force.

So by the definition of dynamic similarity

$$\frac{(F_i)_m}{(F_i)_p} = \frac{(F_v)_m}{(F_v)_p} = \frac{(F_g)_m}{(F_g)_p} = F_r = \text{Force ratio}$$

Que 5.23. **A pipe of dia. 1.5 m is required to transmit an oil of specific gravity 0.9 and viscosity 3×10^{-2} poise at 3000 lps. Tests were conducted on 150 mm dia. pipe using water at 20 °C. Find velocity and rate of flow of model if 'm' water at 20 °C is 0.01 poise.**

AKTU 2018-19, Marks 07

Answer

Given : Diameter of prototype, D_P = 1.5 m, Viscosity of oil, $m_p = 3 \times 10^{-2}$ poise, Discharge for prototype, Q_p = 3000 lit/sec = 3.0 m³/sec, Specific gravity of oil, S_p = 0.9, Density of oil, $\rho_P = S_p \times 1000 = 0.9 \times 1000 = 900$ kg/m³, Diameter of the model, D_m = 15 cm = 0.15 m, Viscosity of water at 20 °C, m_m = 0.01 poise $= 1 \times 10^{-2}$ poise, Density of water, ρ_m = 1000 kg/m³

To Find : Velocity and rate of flow.

1. For pipe flow, the dynamic similarity will be obtained if the Reynold's number in the model and prototype are equal. [For pipe, linear dimension is D]

$$\rho_m v_m D_m / m_m = \rho_P v_P D_P / m_P$$

$$v_m / v_P = \rho_P v_P D_P / \rho_m v_m D_m$$

$$= \frac{900}{1000} \times \frac{1.5}{0.15} \times \frac{1 \times 10^{-2}}{3 \times 10^{-2}} = \frac{900}{1000} \times 10 \times \frac{1}{3} = 3.0$$

2. Velocity in prototype, $$v_P = \frac{3.0}{\frac{\pi}{4}(D_P)^2} = \frac{3.0}{\frac{\pi}{4}(1.5)^2} = 1.698 \text{ m/sec}$$

3. Velocity of fluid in model, $v_m = 3.0 \times v_P = 3.0 \times 1.698 = 5.094$ m/sec

4. Rate of flow through model,

$$Q_m = A_m \times v_m = \pi D_m{}^2 \times v_m / 4 = \pi \times 0.15^2 \times 5.094/4 \text{ m}^3/\text{sec}$$
$$= 0.09 \text{ m}^3/\text{sec} = 0.09 \times 1000 \text{ lit/sec} = 90 \text{ lit/sec}$$

Que 5.24. **In order to predict the performance of a large centrifugal pump, a scale model of one sixth size was made with following specifications. Power = 25 kW, H_{man} = 7 mtr, N = 1000 rpm. If prototype works against 22 m. Calculate its working speed, the power required to derive it and the ratio of flow rates handled by two pumps.**

AKTU 2017-18, Marks 07

Answer

Given : Ratio of linear dimensions of a model and its prototype is equal to = D_m/D_p = 1/6, Speed of model, N_m = 1000 rpm, Head of model, H_m = 7 m, Power of model, P_m = 25 kW, Head of prototype, H_p = 22 m

To Find : Working speed, Power of prototype, Ratio of flow rates of both.

1. Speed of prototype,

$$\left(\frac{\sqrt{H}}{DN}\right)_m = \left(\frac{\sqrt{H}}{DN}\right)_p \Rightarrow N_p = \frac{\sqrt{H_p}}{\sqrt{H_m}} \times \frac{D_m}{D_p} \times N_m$$

$$N_p = \frac{\sqrt{22}}{\sqrt{7}} \times \frac{1}{6} \times 1000 = 295.5 \text{ rpm}$$

2. Power developed by prototype,

$$\left(\frac{P}{D^5N^3}\right)_m = \left(\frac{P}{D^5N^3}\right)_p \Rightarrow P_p = P_m \times \left(\frac{D_p}{D_m}\right)^5 \times \left(\frac{N_p}{N_m}\right)^3$$

$$P_p = 25 \times (6)^5 \times \left(\frac{295.5}{1000}\right)^3 = 5016.13 \approx 5017 \text{ kW}$$

3. Ratio of the flow rates of two pumps,

$$\left(\frac{Q}{D^3N}\right)_m = \left(\frac{Q}{D^3N}\right)_p \Rightarrow \frac{Q_p}{Q_m} = \frac{D_p^3\, N_p}{D_m^3\, N_m} = \left(\frac{D_p}{D_m}\right)^3 \times \frac{N_p}{N_m}$$

$$\frac{Q_p}{Q_m} = 6^3 \times \left(\frac{295.5}{1000}\right) = 63.828$$

Que 5.25. **What is cavitation ? What is its effect on turbine ? How it can be avoided in turbines.**

AKTU 2018-19, Marks 07

Answer

Cavitation in Turbine : The formation, growth and collapse of power filled cavities or bubbles in a following liquid due to local fall in fluid pressure is called cavitation. The hydraulic turbine is affected by cavitation.

Effect of Cavitation on Turbine : The hydraulic turbine is affected by cavitation in following three ways :

1. Roughening of the surface takes place due to loss of material caused by pitting.
2. Vibration of parts is caused due to irregular collapse of cavities.
3. The actual volume of liquid flowing through the machine is reduced causing sudden drop in output and efficiency.

Methods to Avoid Cavitation : The following methods can be used to avoid cavitation :

1. Runner/turbine may be kept under water.
2. Use runner of low specific speed.
3. The cavitation free runner may be designed.
4. Select materials which resist cavitation effect.
5. Cavitation can be reduced by polishing the surface.

Cavitation Factor : The critical value of cavitation factor is given by,

$$\sigma_c = \frac{(H_a - H_v) - H_s}{H}$$

where, H_a = Atmospheric pressure head (m).
H_v = Vapour pressure head (m).
H = Working head of turbine.
H_s = Suction pressure head.

Que 5.26. **What are the three main characteristics of a water turbine ? Define unit power, unit discharge and unit speed. Also sketch constant head curves for Pelton wheel.**

Answer

A. Characteristics of Water Turbine :

1. Characteristic curves are defined as the curves, with the help of which the exact behaviours and performance of the turbine under different working conditions can be known.
2. Following are the important characteristic curves for the turbine :

i. **Main Characteristic Curves or Constant Head Curves :**

a. These are obtained by maintaining a constant head and a constant gate opening (GO) on the turbine.

b. The speed of the turbine is varied by changing load on the turbine.

c. These curves are drawn between the unit quantities. Curves are shown in Fig. 5.26.1.

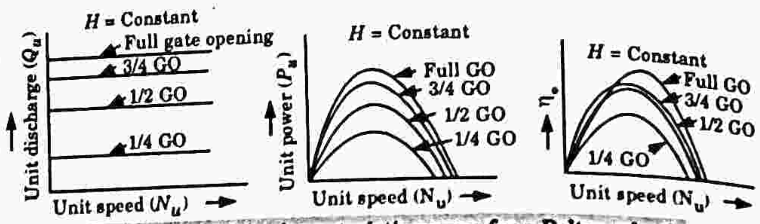


Fig. 5.26.1. Main characteristic curves for a Pelton wheel.

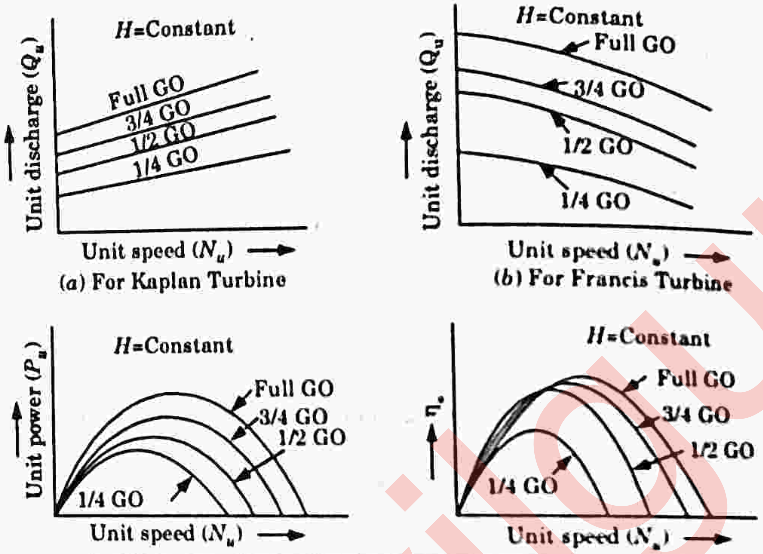


Fig. 5.26.2. Main characteristic curves for reaction turbine.

ii. **Operating Characteristic Curves or Constant Speed Curves :**

a. These are plotted when the speed of the turbine is constant.

b. For operating characteristics N and H are constant and hence the variation of power and efficiency with respect to discharge Q are plotted.

c. These curves are shown in Fig. 5.26.3.

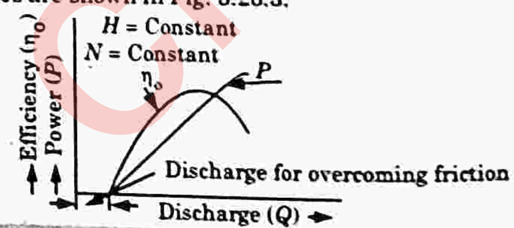


Fig. 5.26.3. Operating characteristic curves.

iii. **Constant Efficiency Curves or Muschel Curves or Iso-efficiency Curves :**

a. These curves are obtained from the speed *vs* efficiency and speed *vs* discharge curves for different gate openings.

b. For a given efficiency from the N_s *vs* h_0 curves, there are two speeds.

c. From the N_u *vs* Q_u curves, corresponding to two values of speeds there are two values of discharge.

d. Hence for a given efficiency there are two values of discharge for a particular gate opening.

e. These curves are shown in Fig. 5.26.4.

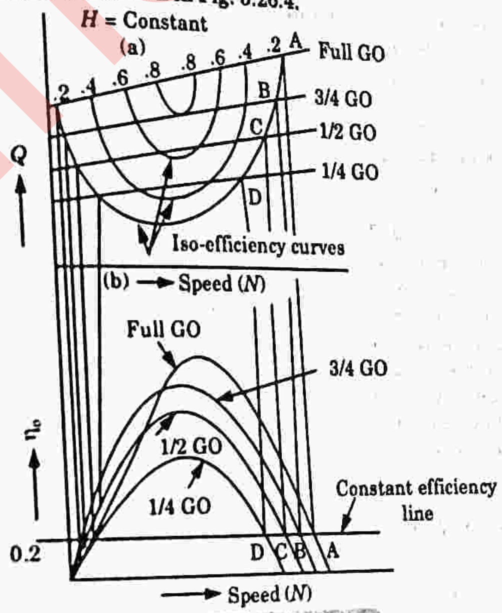


Fig. 5.26.4. Constant efficiency curves.

B. **Unit Speed, Unit Discharge, and Unit Power :** Refer Q. 5.21, Page 5–23B, Unit-5.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Give a brief introduction of rotodynamic machines.

Ans. Refer Q. 5.1, Unit-5.

Q. 2. Define different types of efficiency of hydraulic turbines.

Ans. Refer Q. 5.6, Unit-5.

Q. 3. Design a Pelton wheel which is required to develop 1500 kW, when working under a head of 160 m at a speed of 420 rpm. The overall efficiency may be taken as 85 % and assume other data required.

Ans. Refer Q. 5.11, Unit-5.

Q. 4. Draw neat sketch of various shapes of draft tubes. Also, explain the theory of draft tube.

Ans. Refer Q. 5.17, Unit-5.

Q. 5. Differentiate between impulse and reaction turbine Radial and axial flow turbine.

Ans. Refer Q. 5.18, Unit-5.

Q. 6. An inward flow reaction turbine is required to develop 300 kW at 200 rpm. The active head at the turbine is 18 m. Determine the outside and inside diameters, the inlet and exit angles for the vanes and the exit angle for the guide vanes. Assume the inlet diameter equal to twice the outlet diameter, the hydraulic efficiency as 80 %, the constant radial velocity of flow of 3.6 m³/sec through the runner, the mechanical efficiency as 95 % and the width ratio as 0.1, water leaves the runner radially.

Ans. Refer Q. 5.19, Unit-5.

Q. 7. What is cavitation ? What is its effect on turbine ? How it can be avoided in turbines.

Ans. Refer Q. 5.25, Unit-5.

Q. 8. What are the three main characteristics of a water turbine ? Define unit power, unit discharge and unit speed. Also sketch constant head curves for Pelton wheel.

Ans. Refer Q. 5.26, Unit-5.

☺☺☺

1 UNIT

Flow Through Open Channel (2 Marks Questions)

1.1. Define open channel flow with example.

AKTU 2015-16, Marks 02

Ans. Flow in open channel is defined as the flow of a liquid with a free surface. A free surface is a surface having constant pressure such as atmospheric pressure. Thus a liquid flowing at atmospheric pressure through a passage is known as flow in open channel.

Example : River, canal, etc.

1.2. What do you understand by uniform flow ?

AKTU 2016-17, Marks 01

Ans. Uniform flow is defined as the type of flow in which the velocity at any given time does not change with respect to space.

$$\left(\frac{\partial v}{\partial s}\right)_{t\,=\,\text{constant}} = 0$$

1.3. Define the velocity contour's in open channel flow.

AKTU 2017-18, Marks 02

Ans. A method of measuring stream discharge in which point velocity measurements are translated into average cross-sectional flow velocities by contouring the point velocities; these averages are then multiplied by the areas of the cross sections to give the discharge.

1.4. Define the section factor.

Ans.

1. Section factor is the product of wetted area and square root of the hydraulic depth (for critical flow computation). It is denoted by Z.

$$Z = A\sqrt{D} = A \times \sqrt{\frac{A}{T}} = \left(\frac{A^3}{T}\right)^{1/2}$$

2. Hydraulic radius is the ratio of wetted area (A) to wetted perimeter (P). It is represented by R. Thus,

$$R = \frac{A}{P}$$

1.5. Describe specific energy. **AKTU 2015-16, Marks 02**

Ans. The specific energy of flowing liquid is the energy per unit weight of the liquid with respect to the bottom of the channel.

1.6. Write down the mathematical formula for specific energy.

Ans. The specific energy can be given by, $E = y + v^2/2g = E_p + E_k$

where, E = Specific energy, v = Mean velocity of flow.

y = Depth of flow.

E_p = Potential energy, and E_k = Kinetic energy.

1.7. Define the critical depth.

Ans. The depth of flow occur at minimum specific energy condition is known as critical depth. It is denoted by y_c.

For rectangular channel, critical depth, $y_c = (q_c^2/g)^{1/3}$

1.8. What is kinetic energy correction factor ?

Ans. Kinetic energy correction factor is defined as the ratio of the kinetic energy of the flow per second based on actual velocity across a section to the kinetic energy of the flow per second based on average velocity across the same section. It is denoted by α.

$$\alpha = \frac{\Sigma v^2 \, \Delta A}{V^3 A}$$

1.9. Differentiate between the rigid and mobile boundary channels.

Ans.

S. No.	Rigid Channels	Mobile Boundary Channels
i.	Boundaries of the channel are not deformable.	Boundaries of the channel are deformable.
ii.	There is no scour, erosion or deposition takes place in channel.	There is erosion or deposition takes place.
iii.	**Example :** Lined canals, sewers and non erodible unlined channels etc.	**Example :** Unlined, natural or man made channels in alluvium etc.

1.10. What do you understand by specific force ?

Ans. It is the sum of the momentum of the flow passing through the channel per unit time per unit weight of water and force per unit weight of water

Specific force, $F = \dfrac{Q^2}{Ag} + A\bar{h}$

where, $\bar{h}$ = Depth of the centroid of the area below the surface of the flow.

1.11. What is specific energy curve ?

Ans. It is defined as the curve which shows the variations of specific energy with depth of flow.

1.12. What are the characteristics of specific energy curve ?

Ans. **Characteristics of Specific Energy Curve :**

i. The curve is asymptotic to abscissa and the line OA, *i.e.*, $E = y$. (Refer Fig. 1).

ii. At the point of minimum specific energy, the depth of flow is critical.

iii. For two different depth y_1 and y_2 at same specific energy is obtained and these two depths at same specific energy are called alternate depths.

1.13. Write down the Manning's equation for uniform flow in open channel. **AKTU 2016-17, Marks 02**

Ans. The Manning's equation is given by, $v = \dfrac{1}{n} R^{2/3} S_0^{1/2}$

where, n = Coefficient of roughness.

1.14. State the relation between Manning's constant and Chezy's constant. **AKTU 2015-16, 2018-19; Marks 02**

Ans. Relation between Manning's constant and Chezy's constant is given by, $C = R^{1/6}/n$

1.15. Differentiate between most economical and most efficient channel. **AKTU 2018-19, Marks 02**

Ans.

S. No.	Most Economical Channel	Most Efficient Channel
i.	A section of channel is said to be economical when the cost of construction of the channel is minimum.	Hydraulically most efficient section of a channel can be defined as the one which passes maximum discharge for given area, roughness and bed slope.
ii.	Cost of construction depends upon the excavation and the lining.	Discharge depends on wetted perimeter of section.

1.16. What is critical slope ?

Ans. Critical slope is the slope of a specified channel necessary to have uniform flow of a given discharge with critical depth as the normal depth. Critical slope, $S_c = \dfrac{n^2Q^2}{A_c^2 R_c^{4/3}}$

1.17. Define equivalent roughness.

Ans. Equivalent roughness is a weighted average value for the different roughness coefficient. It is also called composite roughness.

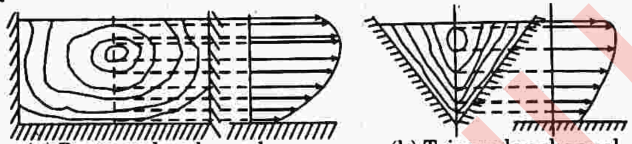
1.18. Write the Horton's formula for equivalent roughness estimation.

Ans. Horton's formula, $n = \dfrac{\left[\sum n_i^{3/2} / P_i\right]^{2/3}}{P^{2/3}}$

where, P = Perimeter of channel.

1.19. Sketch the velocity distribution in rectangular and triangular channels. AKTU 2015-16, 2018-19; Marks 02

Ans.

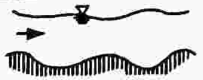
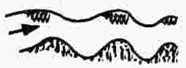


(a) Rectangular channel (b) Triangular channel

Fig. 1. Velocity distribution in open channel.

1.20. Differenciate between the dunes and antidunes bed forms.

Ans.

S.No.	Dunes	Antidunes
i.	Small angular undulation on the bed referred as ripples. An increase of discharge over a rippled bed causes ripples to grow in size with a flat upstream face and step downstream face. They are larger in size than ripples and are called dunes.	If the shear stress in our hypothetical channel is further increased beyond transition phase, the symmetrical sediment wave and the associated standing wave slowly start moving upstream. The waves gradually grow steeper and then break. The bed form at this stage is called antidunes.
ii.	Separation and large energy losses occur.	No separation occurs.
iii.		

1.21. Determine the maximum discharge through a rectangular open channel of area 8 m² with a bed slope of 1/2000. Assume Manning's constant 0.022. AKTU 2017-18, Marks 02

Ans.

Given : Area, $A = 8\ \text{m}^2$, Bed slope $S = \dfrac{1}{2000}$,

Manning's constant, $N = 0.022$

To Find : Maximum discharge

1. For maximum discharge through rectangular channel
   Width of channel,
   $$b = 2d$$
   $$A = b \times d = 2d \times d$$
   $$A = 2d^2$$
   $$8 = 2d^2$$
   Depth of channel, $d = 2$ m
   Width of channel, $b = 2 \times d = 2 \times 2 = 4$ m
2. Hydraulic mean depth, $R = \dfrac{A}{P} = \dfrac{b \times d}{2d + b} = \dfrac{2d \times d}{2d + 2d} = \dfrac{d}{2} = \dfrac{2}{2} = 1\text{m}$
3. Discharge, $Q = \dfrac{1}{N} \times R^{1/6} \times A\sqrt{RS}$
   $$Q = \frac{1}{0.022} \times 1^{1/6} \times 8 \times \sqrt{1 \times \frac{1}{2000}} = 8.13\ \text{m}^3/\text{sec}$$

☺☺☺

2 UNIT

Uniform Flow in Open Channel (2 Marks Questions)

2.1. How can you define the non-uniform flow ?

AKTU 2016-17, Marks 01

Ans. In non-uniform flow or varied flow, the depth of flow and the characteristic changes along the length of the channel.

2.2. Explain GVF and RVF. **AKTU 2018-19, Marks 02**

Ans. There are two types of non-uniform flow, *i.e.*,

i. Gradually varied flow (GVF), in which the change in depth takes place over a long distance.

ii. Rapidly varied flow (RVF), in which the change in depth takes place over a short distance.

2.3 List the assumptions made in the derivation of dynamic equation of gradually varied flow.

AKTU 2015-16, Marks 02

OR

State and discuss the assumptions made in the derivation of the dynamic equation for gradually varied flow.

AKTU 2016-17, Marks 02

Ans. The following assumptions are made in the derivation of GVF equation :

i. The channel is prismatic, *i.e.*, it has a constant shape, size, slope, and alignment.

ii. The pressure distribution over the channel section is hydrostatic.

iii. The slope of the channel is small.

2.4. Briefly explain gradually varied flow.

AKTU 2016-17, Marks 02

Ans. In gradually varied flow, the depth of flow and other characteristics changes along the long length of the channel.

2.5. Give the equation of gradually varied flow.

Ans. Dynamic equation for GVF is given by, $\frac{dy}{dx} = \frac{S_o - S_f}{1 - Q^2T/gA^3}$

2.6. Explain the limitations of gradually varied flow.

Ans. Following are the limitations of GVF :

i. The velocity distribution may not be uniform over cross-section.

ii. The pressure distribution may not be hydrostatic over entire channel section.

iii. The energy and momentum correction factors are not always unity.

iv. The flow is one dimensional, *i.e.*, can only calculate the average cross-sectional water velocity.

2.7. Give the classification of channel slopes.

Ans. Channel slopes are found in following types :

i. Critical slope. ii. Mild slope. iii. Steep slope.

iv. Horizontal slope. v. Adverse slope.

2.8. How can you define the transitional depth ?

Ans. The transitional depth is defined as the depth at which the normal discharge Q_n is equal to the critical discharge Q_c and the slope of the gradually varied flow profile is horizontal. For such a situation,

$$dy/dx = S_0$$

2.9. Discuss the assumptions of the standard step method.

Ans. The basic assumptions of the standard step method are as follows :

i. Steady flow.

ii. Hydrostatic pressure distribution.

iii. One-dimensional analysis.

iv. Small channel slope.

v. Rigid boundary sections.

vi. Constant (averaged) friction slope between adjacent sections.

2.10. Write down the dynamic equation for gradually varied flow in wide rectangular channel. **AKTU 2016-17, Marks 02**

Ans. Dynamic equation for gradually varied flow in wide rectangular channel is given by,

$$\frac{dy}{dx} = S_0\left[\frac{1-(y_0/y)^{\frac{10}{3}}}{1-(y_c/y)^3}\right]$$

2.11. Which profiles occur when dy/dx has negative value ?

Ans. M_2, A_2, H_2, profile occurs when dy/dx has negative value.

2.12. Write the name of the profiles which are not exit.

Ans. A_1, H_1, C_2 profile are not exit.

2.13. Discuss the characteristics of surface profiles.

AKTU 2016-17, Marks 02

OR

Classify the surface profiles in channel.

AKTU 2015-16, Marks 02

OR

What are the classifications of flow profile ?

AKTU 2018-19, Marks 02

Ans.

Table 1. Classification of flow profile.

S.No.	Channel Category	Symbol	Characteristic Condition	Remark
1.	Mild slope	M	$y_0 > y_c$	Subcritical flow at normal depth.
2.	Steep slope	S	$y_c > y_0$	Supercritical flow at normal depth.
3.	Critical slope	C	$y_c = y_0$	Critical flow at normal depth.
4.	Horizontal bed	H	$S_0 = 0$	Cannot sustain uniform flow.
5.	Adverse slope	A	$S_0 < 0$	Cannot sustain uniform flow.

2.14. What are the conditions for existence of S_1, S_2 and S_3 profile on steep slope ?

Ans. For S_1 profile $\Rightarrow y > y_c > y_0$
For S_2 profile $\Rightarrow y_c > y > y_0$
For S_3 profile $\Rightarrow y_c > y_0 > y$

2.15. Give the condition for existence of M_1, M_2 and M_3 profile on mild slope.

Ans. For M_1 profile $\Rightarrow y > y_0 > y_c$
For M_2 profile $\Rightarrow y_0 > y > y_c$
For M_3 profile $\Rightarrow y_0 > y_c > y$

2.16. What do you understand by transition ?

Ans. Transition is the portion of a channel with varying cross-section used for connecting one uniform channel to another.

2.17. What are the functions of channel transition ?

Ans. Functions of channel transition are as follows :
i. Metering of flow.
ii. Dissipation of energy.
iii. Reduction or increase in velocity.
iv. Change in channel section or alignment with minimum energy dissipation, etc.

2.17. What is the back water curve ? **AKTU 2017-18, Marks 02**

Ans. The profile of the rising water on the upstream side of the dam is called back water curve.

☺☺☺

3 UNIT

Rapidly Varied Flow and Open Channel Surge (2 Marks Questions)

3.1. What is hydraulic jump ?

Ans. The hydraulic jump is defined as the sudden and turbulent passage of water from a supercritical state to subcritical state.

3.2. Classify the hydraulic jump.

Ans. The hydraulic jumps are classified into five categories based on Froude number :
i. Undular jump ($1.0 < F_r \le 1.7$).
ii. Weak jump ($1.7 < F_r \le 2.5$).
iii. Oscillating jump ($2.5 < F_r \le 4.5$).
iv. Steady jump ($4.5 < F_r \le 9.0$).
v. Strong or choppy jump ($F_r > 9.0$).

3.3. Write down the Belanger momentum equation.

Ans. Belanger momentum equation is :

$$\frac{y_1}{y_2} = \frac{1}{2}[-1+\sqrt{1+8F_{r2}^2}]$$

3.4. What are the basic assumptions made for the momentum equation formulation of the jump ?

Ans. Following assumptions are made in the analysis of hydraulic jump :
i. The distribution of pressure is hydrostatic.
ii. Momentum correction factor is unity.
iii. Loss of head due to friction at the walls and channel bed is negligible.
iv. The channel is horizontal or it has a small slope. The weight component in direction of flow is neglected.

3.5. Explain the characteristics of the jump.

Ans. Characteristics of the jump are as follows :
i. Length of the jump, L_j, it is given by, $L_j = 6.9\,(y_2 - y_1)$.
ii. The pressure at the toe of the jump and at the end of the jump follows hydrostatic pressure distribution.
iii. The water surface profile of the jump is useful in the efficient design of side walls and the floor of a stilling basin.

3.6. What do you mean by undular jump ?

Ans. The water surface is undulating with a very small ripple on the surface. The sequent depth ratio is very small and E_L/E_1 is practically zero in undular jump. Froude number is found in range of $1.0 < F_r \le 1.7$.

3.7. What are the different characteristics of oscillating jump ?

Ans. This type of jump is characterized by instability of the high velocity flow in the jump which oscillates in a random manner between the bed and the surface. The Froude number is found in range of $2.5 < F_r \le 4.5$

3.8. Give the characteristics of steady jump.

Ans. This type of jump is well established. The roller and jump action is fully developed to cause appreciable energy loss. The energy dissipation ranges from 45 % to 70 %.

3.9. What do you understand by strong or choppy jump ?

Ans. In this jump, the water surface is very rough and choppy. The sequent depth ratio is large and the energy dissipation is very efficient, greater than 70 %. The Froude number is greater than 9.0 in this jump.

3.10. What is weak jump ?

Ans. The surface roller makes its appearance at $F_r = 1.7$ and gradually increases in intensity towards the end *i.e.*, $F_r = 2.5$. The energy dissipation is very small about 5 % at $F_r = 1.7$ and 18 % at $F_r = 2.5$.

3.11. Write down the expression for energy loss in non-rectangular channel.

Ans. Energy loss, $E_L = E_1 - E_2 = y_1 - y_2 + \dfrac{Q^2}{2g}\left(\dfrac{1}{A_1^2} - \dfrac{1}{A_2^2}\right)$

3.12. Give the expression for energy loss in jump on a sloping floor.

Ans. The energy loss can be calculated as,

$$E_L = y_1 \cos\theta + \frac{v_1^2}{2g} + L_j \tan\theta - y_2 \cos\theta - \frac{v_2^2}{2g}$$

where, L_j = Length of jump.

3.13. Write down the uses of hydraulic jump.

Ans. Following are the uses of hydraulic jump :

i. Dissipation of the energy of water falling over a weir or spillway.

ii. Thorough mixing of a chemical in water by introducing it upstream of the jump.

iii. Improvement of the water quality by increasing the dissolved oxygen content through air-entrainment.

3.14. What do you mean by surge ?

Ans. A surge or surge wave is a moving wave front which brings about an abrupt change in depth of flow. A surge is also referred to as moving hydraulic jump and is caused by sudden increase or decrease in flow.

3.15. Define positive surge and negative surge.

OR

Write the types of surge. **AKTU 2017-18, Marks 02**

Ans.

i. A positive surge is one which results in an increase in the depth of flow.

ii. A negative surge is one which results in decrease in the depth of flow.

3.16. Explain deep water waves.

Ans. When the ratio of water depth to wavelength is greater than 0.5, the wave is known as deep water wave. In deep water wave, surface layer of liquid is disturbed.

3.17. What is shallow water wave ?

Ans. The ratio of water depth to wavelength is less than 0.5, the wave is known as shallow water waves. In shallow water waves, entire depth of water is disturbed.

3.18. Define celerity of surge. **AKTU 2015-16, Marks 02**

Ans. The velocity of the surge relative to the initial velocity in the canal is known as celerity of surge.

For downstream surge, celerity, $C = v_w - v_1$

For upstream surge, celerity $C = v_w + v_1$

3.19. Write down the expression for celerity of wave.

Ans. Celerity, $C = \sqrt{\dfrac{g y_2}{2 y_1}(y_2 + y_1)}$

☺☺☺

4 UNIT

Pumps (2 Marks Questions)

4.1 Define the pump.

Ans. A pump may be defined as a mechanical device which when interposed in a pipe line, converts the mechanical energy supplied to it from some external source into hydraulic energy and transfer the same to the liquid through the pipe line, thereby increasing the energy of the flowing liquid.

4.2. What is rotodynamic pump ?

Ans. In the rotodynamic pump, fluid is moved by dynamic action of importing momentum to the fluid, depending on the direction of flow of liquid within the passage of the impeller.

4.3. What do you mean by centrifugal pump ?

Ans. When the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, then the hydraulic machine is called centrifugal pump.

4.4. What is impeller ?

Ans. Impeller is a rotating part provide with a series of backward curved blade or vanes. It is mounted on a shaft which is connected to the shaft of external source of energy, which makes it rotate.

4.5. What are the advantages of centrifugal pumps over reciprocating pumps ?

Ans.

S. No.	Centrifugal Pumps	Reciprocating Pumps
i.	Discharging capacity is very high.	Discharging capacity is comparatively low.
ii.	Maintenance cost of centrifugal pump is low and only periodical check up is sufficient.	Maintenance cost is high.
iii.	It can operate at very high speed.	It has limited speed.

4.6. What is meant by cavitation ? **AKTU 2015-16, Marks 02**

Ans. Cavitation is the phenomenon of formation of vapour bubbles of any flowing liquid in a region, when the pressure at the suction side of the pump drops below the vapour pressure and then sudden collapsing of those bubbles in a resign of high pressure.

4.7. What is priming of centrifugal pump ?

Ans. Priming of centrifugal pump is a process by which the suction pipe, casing of pump, portion of delivery pipe up to delivery valve is completely filled with the liquid from external sources.

4.8. What is the specific speed of centrifugal pump ?

Ans. The specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump which would deliver one cubic meter of liquid per second against a head of one meter.

$$N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}$$

4.9. What is manometric head ?

Ans. This is the head against which a centrifugal pump has to work. It is denoted by H_m and given by,

$$H_m = \frac{V_{w_2} u_2}{g} - \text{Losses}$$

4.10. What do you mean by manometric efficiency ?

Ans. Manometric efficiency is defined as the ratio of the manometric head developed by the pump to the head imported by the impeller to the liquid.

4.11. What is mechanical efficiency ?

Ans. Mechanical efficiency is defined as the ratio of the power actually delivered by the impeller to the power supplied to the shaft by the prime mover or motor.

4.12. What do you understand by overall efficiency ?

Ans. The overall efficiency of the pump is defined as the ratio of the power output from the pump to the power input from the prime mover driving the pump.

4.13. Discuss the effects of cavitation.

Ans. Following are the effects of cavitation :

i. The metallic surfaces are damaged and cavities are formed on the surfaces.

ii. Due to sudden collapsing of vapour bubbles, considerable noise and vibrations are produced.

iii. The efficiency of pump decreases due to cavitation.

4.14. Give the precautions from cavitation in the pumps.

Ans. Following are the precautions from cavitation :

i. Pressure of flowing liquid in any part of the pump should not be allowed to fall below its vapour pressure.

ii. Cavitation resistance material should be used.

4.15. What are the functions of multistage centrifugal pump ?

Ans. Following are the functions of multistage centrifugal pump :

i. To produce a high head.

ii. To discharge a large quantity of liquid.

4.16. What are the functions of a volute casing of a centrifugal pump ?

Ans. The main functions of a volute casing of a centrifugal pump are :

i. To collect water from the periphery of the impeller and to transmit it to the delivery pipe at a constant velocity.

ii. To increase the efficiency of the pump by eliminating the loss of head due to change of velocity.

4.17. What is net positive suction head (NPSH) ?

Ans. It is defined as the net head (in meter) of liquid required to force the liquid into the pump through the suction head.

$$\text{NPSH} = H_{\text{atm}} - \left[H_s + H_{vap} + H_F + \frac{v_s^2}{2g}\right]$$

4.18. What is the coefficient of velocity ?

Ans. It is defined as the ratio of the actual velocity of jet of liquid at vena contracta to the theoretical velocity of jet. It is denoted by C_v and given as,

$$C_v = \frac{v}{\sqrt{2gH}} = 0.95 \text{ to } 0.99$$

4.19. What do you mean by coefficient of contraction ?

Ans. It is defined as the ratio of the area of jet at vena contracta to the area of orifices. It is denoted by C_c and given as,

$$C_c = \frac{\text{Area of jet at vena contracta}}{\text{Area of orifice}} = \frac{a_c}{a}$$

4.20. What is the characteristic curve of a pump ?

Ans. The curve which is drawn to predict the behaviour and performance of a pump under varying conditions is known as characteristic curve of pump.

4.21. Enlist the type of characteristic curves of pump.

Ans. The following three types of characteristic curves are usually prepared for centrifugal pump :

i. Main characteristic curves,

ii. Operating characteristic curves, and

iii. Constant efficiency or Muschel curves.

4.22. Define the terms: speed ratio, flow ratio and jet ratio.

AKTU 2018-19, Marks 02

Ans.

i. **Speed Ratio :** The speed ratio is defined as $= u_1 / \sqrt{2gH}$

where, u_1 = Tangential velocity of wheel at inlet.

ii. **Flow Ratio :** The ratio of the velocity of flow at inlet (V_{f1}) to the velocity given $\sqrt{2gH}$ is known as flow ratio.

iii. **Jet Ratio :** It is defined as the ratio of the pitch diameter (D) of the Pelton wheel to diameter of the jet (d). It is denoted by m and is given by, $m = D / d$

4.23. Which characteristic of pump used for comparing the performance of different pumps ?

Ans. The specific speed is a characteristic of pumps which can be used for comparing the performance of different pumps.

4.24. Why centrifugal pumps is used in series and as well as in parallel ?

Ans. Centrifugal pumps are used in series and parallel as well because if a high head is to be developed, the impeller are connected in series while for discharging large quantity of liquid, the impeller are connected in parallel.

4.25. What is the significance of characteristic curve ?

Ans. The curves are necessary to predict the behaviour and performance of the pump when the pump is working under different flow rate, head and speed.

☺☺☺

5 UNIT Rotodynamic Machines (2 Marks Questions)

5.1. What are rotodynamic machines ?

Ans. The machines whose functioning depends on the principle of fluid dynamics are known as rotodynamic machines.

5.2. Give the classification of rotodynamic machines.

Ans. Different types of rotodynamic machines are as follows :
i. Impulsive hydraulic turbine,
ii. Reaction turbine, and
iii. Rotodynamic pump.

5.3. What is unit speed ?

Ans. It is defined as the speed of a turbine working under a unit head, it is denoted by N_u.

$$N_u = \frac{N}{\sqrt{H}}$$

5.4. Differentiate between inward and outward flow reaction turbines.

Ans.

S. No.	Inward Flow	Outward Flow
i.	Water enters at the outer periphery, flow inward and discharges at the outer periphery.	Water enters at the inner periphery, flows outer and discharges at the outer periphery.
ii.	Centrifugal head imparted is negative.	Centrifugal head imparted is positive.

5.5. What is the function of guide vanes ?

Ans. The basic function of the guide vanes is to convert a part of the pressure energy of the fluid at its entrance to the kinetic energy and then to direct the fluid on the runner blades at the angle appropriate to the design.

5.6. What is draft tube ?

Ans. It is a gradually expanding tube which discharges water, passing through the runner to the tail race. The primary function of the draft tube is to minimize the velocity of discharge, thus, minimizing the loss of kinetic energy at the outlet.

5.7. Explain the functions of air vessels in a reciprocating pump.

AKTU 2016-17, Marks 02

Ans. Following are the various functions of air vessels :
i. It acts as a reservoir that allows continuous flow.
ii. It reduces head loss.
iii. It provides uniform velocity during suction and discharge.

5.8. Describe the surge tank and a forebay and what are their functions.

AKTU 2016-17, Marks 02

Ans. **Surge Tank :** It is a water storage device used as pressure neutralizer in hydropower water conveyance system to resists excess pressure rise and pressure drop conditions. The important functions are as follows :
i. It protects the conduit system from high internal pressures.
ii. It store the water to raise the pressure in pressure drop conditions.

Forebay : A forebay is an artificial pool as water in front of a larger body of water. They may be used upstream of reservoirs to trap sediments and debris in order to keep the reservoir clean.

5.9. What are the advantages of Francis turbine over a Pelton wheel ?

Ans. The Francis turbine claims the following advantages over Pelton wheel :
i. In Francis turbine, the variation in the operating head can be more easily controlled.
ii. In Francis turbine, the ratio of maximum and minimum operating heads can be even two.
ii. The mechanical efficiency of Pelton wheel decrease faster with wear than Francis turbine.

5.10. Explain the efficiency of draft tube.

Ans. The efficiency of draft tube is defined as the ratio of actual conversion of kinetic head into pressure head in the draft tube to the kinetic head at the inlet of the draft tube.

$$\eta_d = \frac{\left(\frac{v_1^2}{2g} - \frac{v_2^2}{2g}\right) - h_f}{v_1^2/2g}$$

5.11. What are the disadvantages of Francis turbine ?

Ans. Francis turbine has the following disadvantages :

i. Water which is not clean can cause very rapid wear in high head Francis turbine.
ii. The overhaul and inspection is much more difficult comparatively.
iii. Cavitation is an ever present danger.
iv. The water hammer effect is more troublesome with Francis turbine.

5.12. Write down the advantages of Kaplan turbine over Francis turbine.

Ans. Kaplan turbine claims following advantages over Francis turbine :

i. For the same power developed, Kaplan turbine is more compact in construction and smaller in size.
ii. Part load efficiency is considerably high.
iii. Low frictional losses.

5.13. Differentiate between Kaplan and Francis turbine.

Ans.

S. No.	Francis Turbine	Kaplan Turbine
i.	Radially inward or mixed flow turbine.	Partially axial flow turbine.
ii.	Medium head turbine.	Low head turbine.
iii.	Flow rate is medium.	Flow rate is large.

5.14. Define axial flow turbine.

Ans. If the water flows parallel to the axis of the rotation of shaft, the turbine is known as axial flow turbine.

5.15. Write the main parts of Kaplan turbines.

AKTU 2017-18, Marks 02

Ans. Following are the main parts of Kaplan turbines :

i. Scroll casing.
ii. Guide vane mechanism.
iii. Draft tube.
iv. Runner blades.

5.16. What are the effects of cavitation on turbine ?

Ans. Following are the effects of cavitation :

i. Power and efficiency of turbine decrease due to cavitation.
ii. Pitting and erosion of metal part.
iii. Distortion of flow pattern.
iv. Structural failure may takes place due to fatigue.

5.17. What is the significance of specific speed ?

Ans. Specific speed plays an important role for selecting the type of the turbine, also the performance of a turbine can be predicted by knowing the specific speed of the turbine.

5.18. Define similitude.

Ans. It is defined as similarity between the model and its prototype in energy respect which means that the model and prototype have similar properties or model and prototype are completely similar.

5.19. What are characteristic curves ?

Ans. These are the curves with the help of which the exact behaviour and performance of the turbine under different working conditions can be known.

5.20. Where Kaplan turbines are suitable ?

Ans. Kaplan turbines are suitable where a large quantity of water of low head is available.

5.21. What are the uses of a draft tube ?

AKTU 2016-17, Marks 02

Ans. The draft tube may be used for the following purposes :

i. It allows the turbine to be set above tail water level, without loss of head, to facilitate inspection and maintenance.
ii. It regions the major portion of the kinetic energy delivered to it from the runner, by diffuse action.

5.22. What assumptions will take in velocity triangles ?

AKTU 2017-18, Marks 02

Ans. Following are the assumptions used to draw velocity triangles :

i. The absolute velocity is radial. Therefore, $v_1 = v_{f1}$ and $v_{w1} = 0$
ii. The blade angle at inlet is such that the blade meets the relative velocity tangentially.
iii. The fluid leaves the impeller with a relative velocity tangential to the blade at outlet.

5.23. Give the range of specific speed values of Kaplan, Francis and Pelton wheel turbine.

AKTU 2015-16, 2018-19; Marks 02

Ans.

S.No.	Specific Speed		Type of Turbine
	(MKS)	(SI)	
1.	10 to 35	8.5 to 30	Pelton wheel with single jet.
2.	35 to 60	30 to 51	Pelton wheel with two or more jets.
3.	60 to 300	51 to 255	Francis turbine.
4.	300 to 1000	255 to 860	Kaplan or propeller turbines.

☺☺☺

B.Tech.

(SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2014-15

HYDRAULICS AND HYDRAULIC MACHINES

Time : 3 Hours **Max. Marks : 100**

Note : 1. Attempt all questions.
2. Assume suitable data, if required.

1. Attempt any **four** parts of the following : (4 × 5 = 20)

a. **Show that for a rectangular channel with given area is most efficient when hydraulic radius is half of the depth of the flow.**

Ans. Refer Q. 1.28, Page 1–24B, Unit-1.

b. **Differentiate between rigid and alluvial channels. Explain dunes and antidunes bed forms with the help of neat sketches.**

Ans. **Difference :** Refer Q. 1.9, 2 Marks Questions, Page SQ–2B, Unit-1.
Dune and Antidune : Refer Q. 1.20, 2 Marks Questions, Page SQ–4B, Unit-1.

c. **Define hydraulic mean radius, hydraulic depth, section factor and most efficient channel cross section.**

Ans. **Hydraulic Mean Radius, Hydraulic Depth and Section Factor :** Refer Q. 1.3, Page 1–4B, Unit-1.
Most Efficient Channel : Refer Q. 1.27, Page 1–24B, Unit-1.

d. **Derive the dynamic equation of gradually varied flow.**

Ans. Refer Q. 2.7, Page 2–10B, Unit-2.

e. **On what factors does the Manning's rugosity coefficient depends ?**

Ans. Refer Q. 1.25, Page 1–23B, Unit-1.

f. **What do you understand by channel of constant velocity ? Derive the relevant formula.**

Ans. Refer Q. 1.17, Page 1–17B, Unit-1.

2. Attempt any **two** parts of the following : (2 × 10 = 20)

a. **A wide rectangular channel carries a flow of 2.75 m^3/sec per meter width, the depth of flow being 1.5 m. Calculate the rise of the floor level required to produce a critical flow condition. What is the corresponding fall in surface level ?**

Ans. Refer Q. 2.2, Page 2–5B, Unit-2.

b. **What do you understand by specific energy for a flow in open channel ? Draw the specific energy diagram and describe its various characteristics.**

Ans. Refer Q. 1.7, Page 1–9B, Unit-1.

c. **Define conveyance of a channel. Find the discharge in a trapezoidal channel with a bed width of 10 m, side slope 1 : 1 and depth of flow of 0.2 m under uniform flow condition. Bed slope is 1×10^{-4} and Manning's roughness coefficient 0.025. Also find Chezy's coefficient at this depth.**

Ans. Refer Q. 1.22, Page 1–21B, Unit-1.

3. Attempt any **two** parts of the following : (2 × 10 = 20)

a. **Sketch the GVF profile produced on**

i. **Mild slope,**

ii. **Steep slope, and**

iii. **Critical slope.**

Ans. Refer Q. 2.14, Page 2–17B, Unit-2.

b. **A rectangular channel with a bottom width of 4.0 and a bottom slope of 0.0008 has a discharge of 1.50 m^3/sec. In gradually varies flow in this channel, the depth at certain location is found to be 0.30 m. Assuming $n = 0.016$, determine the type of GVF profile.**

Ans. Refer Q. 2.15, Page 2–19B, Unit-2.

c. **Derive the dynamic equation of GVF, state its various assumptions, also give the limitations of GVF.**

Ans. **Derivation :** Refer Q. 2.7, Page 2–10B, Unit-2.
Assumptions and Limitations : Refer Q. 2.6, Page 2–9B, Unit-2.

4. Attempt any **two** parts of the following : (2 × 10 = 20)

a. **What do you understand by term hydraulic jump ? Discuss the classification of hydraulic jump and practical applications of hydraulic jump.**

Ans. Refer Q. 3.1, Page 3–2B, Unit-3.

b. **A horizontal rectangular channel of constant width is fitted with a sluice gate. When the sluice gate is opened, water issues with a velocity of 6 m/sec and depth of 0.5 m at the vena contracta. Determine whether a hydraulic jump will form or not. If so, calculate the energy dissipated.**

Ans. Refer Q. 3.7, Page 3–7B, Unit-3.

c. **Hydraulic jump is sometimes used as energy dissipator at the toe of the spillway of a dam, why ? Discuss different ways for obtaining the hydraulic jump. Prove that relative height of the jump, depend only on flow corresponding supercritical conditions Froude number.**

Ans. **Energy Dissipator :** Refer Q. 3.15, Page 3–18B, Unit-3.

Obtaining of Hydraulic Jump : Refer Q. 3.16, Page 3–18B, Unit-3.

Derivation : Refer Q. 3.4, Page 3–5B, Unit-3.

5. Attempt any **two** parts of the following : (2 × 10 = 20)

a. i. **Differentiate between impulse and reaction turbine, radial and axial flow turbine.**

Ans. Refer Q. 5.18, Page 5–19B, Unit-5.

ii. **Write a note on characteristics curves for rotodynamic pumps.**

Ans. Refer Q. 4.25, Page 4–30B, Unit-4.

b. **Draw neat sketch of various shapes of draft tubes. Also, explain the theory of draft tube.**

Ans. Refer Q. 5.17, Page 5–17B, Unit-5.

c. **A Pelton wheel turbine is to be designed for the following specifications :**
Shaft power = 11,775 kW
Head = 400 m, speed = 750 rpm
Overall efficiency = 86 %,
Jet diameter to wheel diameter ratio is not to exceed one sixth (1/6). Find.
i. **The wheel diameter,**
ii. **Diameter of the jet, and**
iii. **The number of jets required.**
Assume K_{v1} = 0.985 and K_{u1} = 0.45

Ans. Refer Q. 5.9, Page 5–9B, Unit-5.

☺☺☺

B. Tech.
(SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2015-16
HYDRAULICS AND HYDRAULIC MACHINE

Time : 3 Hours **Max. Marks : 100**

SECTION - A

Attempt **All** parts. All parts carry equal marks.

1. **Write answer of each part in short.** (10 × 2 = 20)

a. **Define open channel flow with example.**

Ans. Refer Q. 1.1, 2 Marks Questions, Page SQ–1B, Unit-1.

b. **Describe specific energy.**

Ans. Refer Q. 1.5, 2 Marks Questions, Page SQ–2B, Unit-1.

c. **State the relation between Manning's constant and Chezy's constant.**

Ans. Refer Q. 1.14, 2 Marks Questions, Page SQ–3B, Unit-1.

d. **Sketch the velocity distribution in rectangular and triangular channels.**

Ans. Refer Q. 1.19, 2 Marks Questions, Page SQ–4B, Unit-1.

e. **Classify the surface profiles in channel.**

Ans. Refer Q. 2.13, 2 Marks Questions, Page SQ–7B, Unit-2.

f. **List the assumptions made in the derivation of dynamic equation of gradually varied flow.**

Ans. Refer Q. 2.3, 2 Marks Questions, Page SQ–6B, Unit-2.

g. **Hydraulic jump is sometimes used as energy dissipator at the toe of the spillway of a dam. Why ?**

Ans. Refer Q. 3.15, Page 3–18B, Unit-3.

h. **What is meant by cavitations ?**

Ans. Refer Q. 4.6, 2 Marks Questions, Page SQ–12B, Unit-4.

i **Define celerity of the surge.**

Ans. Refer Q. 3.18, 2 Marks Questions, Page SQ–11B, Unit-3.

j. **Give the range of specific speed values of Kaplan, Francis turbine and Pelton wheels.**

Ans. Refer Q. 5.23, 2 Marks Questions, Page SQ–19B, Unit-5.

SECTION - B

2. Attempt any five questions from this section. (5 × 10 = 50)

a. **Classify the following open-channel flow situations :**

i. **Flow from a sluice gate.**
ii. **Flow in a main irrigation canal.**
iii. **A river during flood.**

Ans. Refer Q. 2.13, Page 2–15B, Unit-2.

b. **Show that in rectangular channel maximum discharge occurs when the flow is critical for a given value of specific energy.**

Ans. Refer Q. 1.14, Page 1–15B, Unit-1.

c. **Derive an expression for the discharge through a channel by Chezy's formula.**

Ans. Refer Q. 1.17, Page 1–17B, Unit-1.

d. **The width of a horizontal rectangular channel is reduced from 3.5 m to 2.5 m and the floor is raised by 0.25 m in elevation at a given section. At the upstream section, the depth of flow is 2.0 m and the kinetic energy correction factor α is 1.15. If the drop in the water surface elevation at the contraction is 0.20 m, calculate the discharge if (*a*) the energy loss is neglected, and (*b*) the energy loss is one-tenth of the upstream velocity head. [The kinetic energy correction factor at the contracted section may be assumed to be unity.]**

Ans. Refer Q. 2.3, Page 2–6B, Unit-2.

e. **What is critical depth in open-channel flow ? For a given average flow velocity, how is it determined ?**

Ans. Refer Q. 1.11, Page 1–13B, Unit-1.

f. **A trapezoidal channel B = 3.0 m, m = 1.50, n = 0.025 and S_0 = 0.0005 takes off from a reservoir with free inlet. The reservoir elevation is 7.0 m above the channel bed at the inlet. Calculate the discharge in the channel by neglecting entrance losses.**

Ans. Refer Q. 1.23, Page 1–21B, Unit-1.

g. **Explain the terms :**
i. **Uniform flow in an open channel.**

Ans. Refer Q. 1.17, Page 1–17B, Unit-1.

ii. **Reaction turbine.**

Ans. Refer Q. 5.12, Page 5–12B, Unit-5.

h. **A spillway discharge a flood flow at a rate of 7.75 m³/sec per metre width. At the downstream horizontal apron the depth of flow was found to be 0.50 m. What tail water depth is needed to form a hydraulic jump ? If a jump is formed, find its (*a*) type, (*b*) length, (*c*) head loss, (*d*) energy loss as a percentage of the initial energy, and (*e*) profile.**

Ans. Refer Q. 3.8, Page 3–8B, Unit-3.

SECTION – C

Note : Attempt any two questions from this section. **(2 × 15 = 30)**

3. a. **Draw neat sketches of various shapes of draft tubes.** **(5)**

Ans. Refer Q. 5.17, Page 5–17B, Unit-5.

b. **In a Pelton wheel has a mean bucket speed of 10 m/sec. With a jet of water flowing at the rate of 0.7 m³/sec and a head of 30 m. If the buckets deflects the jet through an angle of 160° calculate the power given by water to the runner and hydraulic efficiency of the turbine. Assume the coefficient of velocity 0.98.** **(10)**

Ans. Refer Q. 5.8, Page 5–8B, Unit-5.

4. a. **A compound channel is symmetrical in cross section and has the following geometric properties : Main channel : Trapezoidal cross section, bottom width = 15.0 m, side slopes = 1.5 H : 1 V, bank full depth = 3.0 m, Manning's coefficient = 0.03, longitudinal slope = 0.0009, Flood plains : width = 75 m, side slope = 1.5 H : 1 V, Manning's coefficient = 0.05, longitudinal slope = 0.0009. Compute the uniform flow discharge for a flow with total depth of 4.2 m by using DCM with either (*i*) diagonal interface, or (*ii*) vertical interface procedures.** **(12)**

Ans. **Compound Channel and Diagonal Interface Procedure :** Refer Q. 1.37, Page 1–34B, Unit-1.
Vertical Interface Procedure : Refer Q. 1.36, Page 1–32B, Unit-1.

b. **A triangular channel with an apex angle of 75° carries a flow of 1.2 m³/sec at a depth of 0.80 m. If the bed slope is 0.009, find the roughness coefficient of the channel.** **(3)**

Ans. Refer Q. 1.26, Page 1–23B, Unit-1.

5. a. **There are three main categories of dynamic pumps. List and define them.** **(6)**

Ans. Refer Q. 4.13, Page 4–18B, Unit-4.

b. **A centrifugal pump is running at 1000 rpm. The outlet vane angle of the impeller is 45° and velocity of flow at outlet is 2.5 m/sec. The discharge through the pump is 200 lit/sec when the pump is working against a total head of 20 m. If the manometric efficiency of the pump is 80 %, determine :**
i. **Diameter of the impeller (outside diameter).** **(9)**
ii. **Width of the impeller at outlet.**

Ans. Refer Q. 4.27, Page 4–33B, Unit-4.

☺☺☺

B. Tech.

(SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2016-17

HYDRAULICS AND HYDRAULIC MACHINE

Time : 3 Hours **Max. Marks : 100**

Note : Be precise in your answer. In case of numerical problem assume data wherever not provided.

SECTION-A

1. **Explain the following :** (10 × 2 = 20)

a. **What do you understand by uniform and non-uniform flow in the case of channels ?**

Ans. **Uniform Flow :** Refer Q. 1.2, 2 Marks Questions, Page SQ–1B, Unit-1.

Non-uniform Flow : Refer Q. 2.1, 2 Marks Questions, Page SQ–6B, Unit-2.

b. **State and discuss the assumptions made in the derivation of the dynamic equation for gradually varied flow.**

Ans. Refer Q. 2.3, 2 Marks Questions, Page SQ–6B, Unit-2.

c. **Write down the Manning's equation for uniform flow in open channel.**

Ans. Refer Q. 1.13, 2 Marks Questions, Page SQ–3B, Unit-1.

d. **Briefly explain in gradually varied flow.**

Ans. Refer Q. 2.4, 2 Marks Questions, Page SQ–6B, Unit-2.

e. **Write down the dynamic equation for gradually varied flow in wide rectangular channel.**

Ans. Refer Q. 2.10, 2 Marks Questions, Page SQ–7B, Unit-2.

f. **Discuss the characteristics of surface profiles.**

Ans. Refer Q. 2.13, 2 Marks Questions, Page SQ–7B, Unit-2.

g. **Explain the working principles of reciprocating pump.**

Ans. Refer Q. 4.7, Page 4–13B, Unit-4.

h. **Explain the functions of air vessels in a reciprocating pump.**

Ans. Refer Q. 5.7, 2 Marks Questions, Page SQ–17B, Unit-5.

i. **What are the uses of a draft tube ?**

Ans. Refer Q. 5.21, 2 Marks Questions, Page SQ–19B, Unit-5.

j. **Describe the surge tank and a forebay and what are their functions.**

Ans. Refer Q. 5.8, 2 Marks Questions, Page SQ–17B, Unit-5.

SECTION-B

2. **Attempt any five of the following questions :** (5 × 10 = 50)

a. **A trapezoidal channel with a base width of 6 m and side slopes of 2 horizontal to 1 vertical conveys water at 17 m^3/sec with a depth of 1.5 m. Is the flow situation sub or super critical ?**

Ans. Refer Q. 1.16, Page 1–16B, Unit-1.

b. **State the conditions under which the rectangular section of an open channel will be most economical. Derive these conditions.**

Ans. Refer Q. 1.28, Page 1–24B, Unit-1.

c. **At what height from water surface a centrifugal pump may be installed in the following case to avoid cavitation : atmospheric pressure 101 kPa; vapour pressure 2.34 kPa; inlet and other losses in suction pipe 1.55 m; effective head of pump 52.5 m; and cavitation parameter σ = 0.118.**

Ans. Refer Q. 4.19, Page 4–25B, Unit-4.

d. **Show that the maximum inertia head in a reciprocating pump without air vessel is given by,**

$$H_a = \frac{l}{g} \times \frac{A}{a} \omega^2 r$$

Ans. Refer Q. 4.12, Page 4–16B, Unit-4.

e. **What is Chezy's formula ? How is it derived ?**

Ans. Refer Q. 1.17, Page 1–17B, Unit-1.

f. **Show that for a trapezoidal channel of given area of flow. The condition of maximum flow requires that hydraulic mean depth is equal to one-half the depth of flow.**

Ans. Refer Q. 1.30, Page 1–26B, Unit-1.

g. **A rectangular channel 10 m wide is laid with a break in its bottom slope from 0.01 to 0.0064. If it carries 125 m^3/sec, determine the nature of the surface profile and compute its length. Take n = 0.015.**

Ans. Refer Q. 2.18, Page 2–22B, Unit-2.

h. **Explain with a neat sketch, the construction details and working principles of a centrifugal pump.**

Ans. Refer Q. 4.15, Page 4–21B, Unit-4.

SECTION-C

Attempt any two of the following questions : (2 × 15 = 30)

3. a. **A trapezoidal channel has a bottom width of 6 m and side slopes of 1 : 1. The depth of flow is 1.5 m at a discharge of 15 m^3/sec. Determine the specific energy. If the critical depth is 0.9 m, discuss the type of flow corresponding to the critical depth.**

Ans. Refer Q. 2.5, Page 2–8B, Unit-2.

b. For a hydraulic jump in a horizontal triangular channel show that

$$3F_{r1}^2 = \frac{\gamma^2(\gamma^2-1)}{\gamma^2-1}$$

where, $F_{r1}^2 = \left(\frac{v_1^2}{gy_1}\right)$ **and,** $\gamma = \left(\frac{y_2}{y_1}\right)$

Ans. Refer Q. 3.5, Page 3–6B, Unit-3.

4. a. A horizontal rectangular channel 4 m wide carries a discharge of 16 m³/sec. Determine whether a jump may occur at an initial depth of 0.5 m or not. If a jump occurs determine the sequent depth to this initial depth. Also determine the energy loss in the jump.

Ans. Refer Q. 3.9, Page 3–10B, Unit-3.

b. Design a Pelton wheel which is required to develop 1500 kW, when working under a head of 160 m at a speed of 420 rpm. The overall efficiency may be taken as 85 % and assume other data required.

Ans. Refer Q. 5.11, Page 5–11B, Unit-5.

5. An inward flow reaction turbine is required to develop 300 kW at 200 rpm. The active head at the turbine is 18 m. Determine the outside and inside diameters, the inlet and exit angles for the vanes and the exit angle for the guide vanes. Assume the inlet diameter equal to twice the outlet diameter, the hydraulic efficiency as 80 %, the constant radial velocity of flow of 3.6 m³/sec through the runner, the mechanical efficiency as 95 % and the width ratio as 0.1, water leaves the runner radially.

Ans. Refer Q. 5.19, Page 5–21B, Unit-5.

☺☺☺

B. Tech.

(SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2017-18

HYDRAULICS AND HYDRAULIC MACHINE

Time : 3 Hours **Max. Marks : 100**

Note : Attempt **all** sections. If require any missing data; then choose suitably.

SECTION-A

1. Attempt **all** questions in brief : **(7 × 2 = 14)**

a. Define different types of flow.

Ans. Refer Q. 1.2, Page 1–3B, Unit-1.

b. Determine the maximum discharge through a rectangular open channel of area 8 m² with a bed slope of 1/2000. Assume Manning's constant 0.022.

Ans. Refer Q. 1.21, 2 Marks Questions, Page SQ–5B, Unit-1.

c. Define the velocity contour's in open channel flow.

Ans. Refer Q. 1.3, 2 Marks Questions, Page SQ–1B, Unit-1.

d. What is the back water curve ?

Ans. Refer Q. 2.17, 2 Marks Questions, Page SQ–8B, Unit-2.

e. Write the types of surge.

Ans. Refer Q. 3.15, 2 Marks Questions, Page SQ–11B, Unit-3.

f. What assumptions will take in velocity triangles ?

Ans. Refer Q. 5.22, 2 Marks Questions, Page SQ–19B, Unit-5.

g. Write the main parts of Kaplan turbines.

Ans. Refer Q. 5.15, 2 Marks Questions, Page SQ–18B, Unit-5.

SECTION-B

2. Attempt any **three** part of the following : **(3 × 7 = 21)**

a. Uniform flow occurs at a depth of 1.5 m in a long rectangular channel 3 m wide and laid to a slope of 0.0009. If Manning's n = 0.015. Calculate (a) Maximum height of hump on the floor to produce critical depth (b) The width of contraction which will produce critical depth without increasing the upstream depth of flow.

Ans. Refer Q. 2.4, Page 2–7B, Unit-2.

b. In an open channel, the Froude number F remains constant at all depths. If the specific energy E is constant show that

$$\frac{T}{B} = \left(\frac{E}{E-y}\right)^{\left(\frac{1+F^2}{2}\right)}$$

Ans. Refer Q. 3.11, Page 3–12B, Unit-3.

c. Prove that hydraulically most efficient trapezoidal section is half of regular hexagon.

Ans. Refer Q. 1.30, Page 1–26B, Unit-1.

d. Integrate the differential equation of GVF for a horizontal channel to get the profile equation as

$$x = \frac{h_c}{S_c}\left[\frac{\left(\frac{h}{h_c}\right)^{N-M+1}}{N-M+1} - \frac{\left(\frac{h}{h_c}\right)^{N+1}}{N+1}\right] + \text{Constant.}$$

Ans. Refer Q. 2.12, Page 2–14B, Unit-2.

e. What is NPSH of centrifugal pump ? How it is related to cavitation in pump ?

Ans. Refer Q. 4.21, Page 4–27B, Unit-4.

SECTION-C

3. Attempt any one part of the following : (1 × 7 = 7)

a. An open channel to be made of concrete is to be designed to carry 1.5 m³/s at a slope of 0.00085. Find the most efficient cross section for (a) Rectangular section (b) Trapezoidal section (c) Semicircular section.

Ans. Refer Q. 1.33, Page 1–29B, Unit-1.

b. Define the following with formula (a) Kinetic energy correction factor (b) Momentum correction factor.

Ans. Refer Q. 4.2, Page 4–3B, Unit-4.

4. Attempt any one part of the following : (1 × 10 = 10)

a. Using basic differential equation of GVF show that dh/dx is positive for S_1, M_3 and S_3 profiles.

Ans. Refer Q. 2.7, Page 2–10B, Unit-2.

b. How you will define transitions between sub critical flow and super critical flow ? Also draw the diagram.

Ans. Refer Q. 2.17, Page 2–21B, Unit-2.

5. Attempt any one part of the following : (1 × 7 = 7)

a. A rectangular channel carrying a super critical stream is to be provided with a hydraulic jump type of energy dissipater. It is desired to have an energy loss of 5 m in hydraulic jump when inlet Froude's number is 8.5. What are the sequent depths of this jump ?

Ans. Refer Q. 3.13, Page 3–16B, Unit-3.

b. Derive the relation between velocity and depths of flow where positive surges moving upward.

Ans. Refer Q. 3.18, Page 3–20B, Unit-3.

6. Attempt any one part of the following : (1 × 7 = 7)

a. In order to predict the performance of a large centrifugal pump, a scale model of one sixth size was made with following specifications. Power = 25 kW, H_{man} = 7 mtr, N = 1000 rpm. If prototype works against 22 m. Calculate its working speed, the power required to derive it and the ratio of flow rates handled by two pumps.

Ans. Refer Q. 5.24, Page 5–27B, Unit-5.

b. Define cavitation. And what precautions taken against cavitation.

Ans. Refer Q. 4.23, Page 4–28B, Unit-4.

7. Attempt any one part of the following : (1 × 7 = 7)

a. A pelton wheel is to be designed for the following specification. Shaft power = 11722 kW, Head = 380 mtr, speed = 750 rpm, η_0 = 86 % jet diameter (d) not to exceed one-sixth of wheel diameter. Determine (i) The wheel diameter (ii) Number of jet required (iii) Diameter of jet, take velocity ratio K_{v1} = 0.985 and speed ratio K_{u1} = 0.45.

Ans. Refer Q. 5.10, Page 5–10B, Unit-5.

b. Define different types of efficiency of hydraulic turbines.

Ans. Refer Q. 5.6, Page 5–7B, Unit-5.

☺☺☺

B. Tech.
(SEM. IV) EVEN SEMESTER THEORY EXAMINATION, 2018-19
HYDRAULICS AND HYDRAULIC MACHINES

Time : 3 Hours **Max. Marks : 70**

Note : Attempt all sections. If require any missing data; then choose suitably.

Section-A

1. Attempt all questions in brief. **(2 × 7 = 14)**

a. **State the relation between Manning's constant and Chezy's constant.**

Ans. Refer Q. 1.14, 2 Marks Questions, Page SQ–3B, Unit-1.

b. **Explain GVF and RVF.**

Ans. Refer Q. 2.2, 2 Marks Questions, Page SQ–6B, Unit-2.

c. **What are the classifications of flow profile ?**

Ans. Refer Q. 2.13, 2 Marks Questions, Page SQ–7B, Unit-2.

d. **Differentiate between most economical and most efficient channel.**

Ans. Refer Q. 1.15, 2 Marks Questions, Page SQ–3B, Unit-1.

e. **Sketch the velocity distribution in rectangular and triangular channels.**

Ans. Refer Q. 1.19, 2 Marks Questions, Page SQ–4B, Unit-1.

f. **Define the terms: speed ratio, flow ratio and jet ratio.**

Ans. Refer Q. 4.22, 2 Marks Questions, Page SQ–15B, Unit-4.

g. **Give the range of specific speed values of Kaplan, Francis and Pelton wheel turbine.**

Ans. Refer Q. 5.23, 2 Marks Questions, Page SQ–19B, Unit-5.

Section-B

2. Attempt any three of the following : **(7 × 3 = 21)**

a. **What is a draft tube ? Write neat sketch, list the different types of draft tube.**

Ans. Refer Q. 5.17, Page 5–17B, Unit-5.

b. **Derive the condition for most efficient trapezoidal channel section for uniform flow.**

Ans. Refer Q. 1.30, Page 1–26B, Unit-1.

c. **A rectangular channel carries water at the rate of 400 litres/sec when bed slope is 1 in 2000. Find the economical dimensions of the channel if $C = 50$.**

Ans. Refer Q. 1.34, Page 1–31B, Unit-1.

d. **Explain the specific energy concept and prove the critical flow condition for all type of channel.**

Ans. Refer Q. 1.9, Page 1–11B, Unit-1.

e. **Differentiate between single stage and multistage pumps.**

Ans. Refer Q. 4.9, Page 4–14B, Unit-4.

Section-C

3. Attempt any one part of the following : **(7 × 1 = 7)**

a. **A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 rpm works against a total head of 40 m. The velocity of the flow through the impeller is constant and is equal is 2.5 m/sec. The vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 500 mm and width at outlet is 50 mm, determine.**

i. **Vane angle at inlet.**

ii. **Work done by impeller on water per second.**

iii. **Manometric efficiency.**

Ans. Refer Q. 4.28, Page 4–34B, Unit-4.

b. **What is reciprocating pump ? Describe the principle and working of a reciprocating pump with a neat sketch.**

Ans. Refer Q. 4.7, Page 4–13B, Unit-4.

4. Attempt any one part of the following : **(7 × 1 = 7)**

a. **Derive an expression for depth of hydraulic jump in terms of upstream Froude's number.**

Ans. Refer Q. 3.4, Page 3–5B, Unit-3.

b. **A rectangular channel is 20 m wide and carries discharge of 65 m³/sec. It is laid at a slope of 0.0001. At a certain section along the channel length, the depth of flow is 2.0 m. How far will be the depth be 2.60 m ? Take $n = 0.02$.**

Ans. Refer Q. 2.21, Page 2–28B, Unit-2.

5. Attempt any one part of the following : **(7 × 1 = 7)**

a. Prove that the discharge over a spillway is given by the relation, $Q = VD^2 f\left[\sqrt{gD} / V,\ H / D\right]$

Ans. Refer Q. 3.24, Page 3–26B, Unit-3.

b. A jet of water of 40 mm diameter strikes a hinged square plate at its centre, with a velocity of 20 m/sec. The plate is deflected through an angle of 30°. Find the weight of the plate. If the plate is not allowed to swing, what will be the force required at the lower edge of the plate to keep the plate in vertical position.

Ans. Refer Q. 4.4, Page 4–7B, Unit-4.

6. Attempt any **one** part of the following : (7 × 1 = 7)

a. What is cavitation ? What is its effect on turbine ? How it can be avoided in turbines.

Ans. Refer Q. 5.25, Page 5–27B, Unit-5.

b. A channel is 2 m width at bottom, the length of each sloping side is 1.95 m. The width of water surface is 5.5 m. The flow depth is 1.2 m and bed slope is 1 in 5280. What is the discharge per minute ? Take value of C = 34.6.

Ans. Refer Q. 1.24, Page 1–22B, Unit-1.

7. Attempt any **one** part of the following : (7 × 1 = 7)

a. Sketch the GVF profiles produced on :

i. Steep Slope, and ii. Critical slope.

Ans. Refer Q. 2.14, Page 2–17B, Unit-2.

b. A pipe of dia. 1.5 m is required to transmit an oil of specific gravity 0.9 and viscosity 3×10^{-2} poise at 3000 lps. Tests were conducted on 150 mm dia. pipe using water at 20 °C. Find velocity and rate of flow of model if 'μ' water at 20 °C is 0.01 poise.

Ans. Refer Q. 5.23, Page 5–26B, Unit-5.

☺☺☺